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# A RANDOM MODEL FOR MINERAL LIBERATION BY SIZE REDUCTION

by R. L. Wiegel and K. Li

*This article proposes a physical model for an idealized binary mineral system and for the process of liberation by size reduction. Based on this model, equations are derived relating the amount of each component liberated as a function of the ratio of the volumetric abundance of the two mineral components and the ratio of the mineral grain size to particle size. A definite relationship is shown to exist between the mineral concentration within a locked particle and the frequency of occurrence of such a particle. These relationships are used to obtain theoretical expressions for the concentration-recovery diagram.*

Physical mineral beneficiation processes are based on breaking a mineral aggregate to liberate the dissimilar mineral grains and then making use of their physical properties to separate the mixture into its components. The effectiveness of the process is therefore dependent on how well the minerals are liberated and how efficiently they are separated.

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Although a substantial amount of work is being done to improve our understanding of mineral separation processes, the study of mineral liberation itself has been almost completely neglected. The purpose of this theoretical study is to shed some light on the liberation phenomenon by: 1) proposing a physical model for an ideal binary mineral system, 2) treating mathematically the fracture of this system into liberated and locked particles, and 3) deriving an expression for the liberation characteristics of this system. The results of this study are useful in understanding the effect and importance of various parameters in liberation, and provide a basis for comparison with real mineral systems.

## 'RANDOM' MODEL

It is recognized that liberation can be caused by detachment which results from fractures occurring at the mineral grain boundaries, or by size reduction which restricts the occurrence of dissimilar mineral locking to a portion of the resulting particles. The detachment effect is dependent on the relative strengths of the mineral grains and the bonds between them, and must be treated empirically for each mineral system. The size reduction effect, however, can be treated analytically if certain assumptions are made concerning the shape and geometrical arrangement of the mineral grains.

Gaudin [1] proposed a model for the fracture of an ideal binary mineral system to explain the size reduction effect, and derived equations relating the fraction of each mineral liberated to the ratio of mineral grain size to particle size, and to the ratio of the volumetric abundance of the two mineral species. The usefulness of the model, however, is restricted by geometric limitations imposed by an assumption regarding mineral grain arrangement, which results in discrete equations for integer values of the volumetric abundance ratio. The equations also are valid only for grain size to particle size ratios greater than unity, and give no information about the distribution of the mineral species in the locked particles.

The model for an ideal binary mineral system proposed in this study differs from Gaudin's model only in the arrangement of the mineral grains. The model treated here is based on a completely random arrangement of mineral grains, while Gaudin chose to place the grains of the least abundant mineral as far apart as possible. From a practical standpoint, the random model describes more realistically the haphazard appearance of a real mineral system, and it implies the existence of a mineral grain size distribution – due to clusters of single grains. From a mathematical standpoint, the random model gives equations which are continuous with respect to the abundance ratio, and valid for all values of the size ratio. It also yields information concerning the distribution of mineral species in the locked particles.

The random model is based on the following assumptions: 1) the grains of both mineral species are cubic and are of the same uniform size ($a$) (Fig. 1);

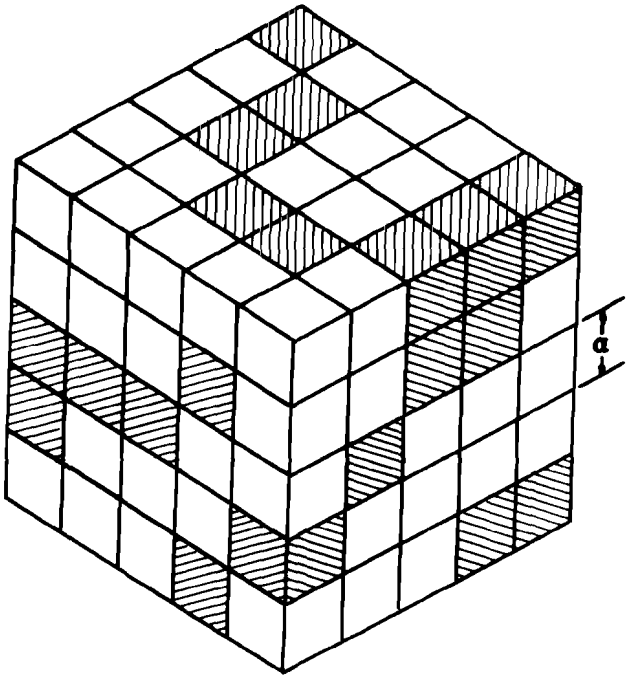



*Fig. 1 – Random arrangement of mineral grains in a binary mineral aggregate.*

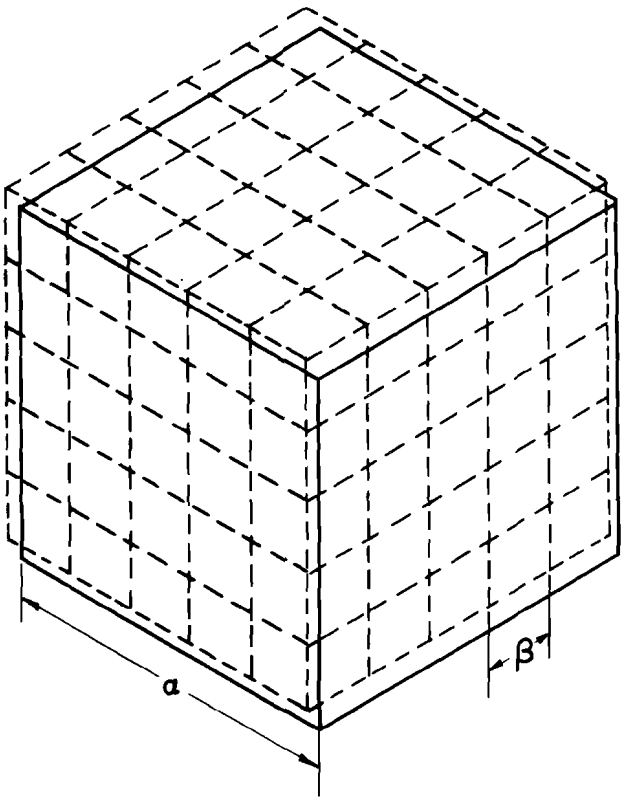



*Fig. 2 – Fracture of a single mineral grain (solid line) into particles (dotted line) of uniform size.*

2) the grains are aligned in the mineral aggregate in a lattice-like arrangement so that grain surfaces form continuous planes; 3) the grains of the two mineral species are randomly arranged throughout the aggregate; 4) by size reduction, the aggregate is broken into particles of uniform size ($\beta$) by a cubic fracture lattice which is superimposed randomly on the aggregate parallel to the grain lattice (Fig. 2).

**Probability of Liberated and Locked Particles – Size Ratio Greater Than Unity:** Consider an aggregate of cubic grains of the same size $a$ of mineral species $A$ and $B$, which are aligned and arranged as described by the random model. When the aggregate is subjected to a random fracture lattice parallel to the grain lattice (Fig. 2), producing smaller cubic particles of size $\beta$, the average number of particles from each grain will be $k^3$, where $k$ is the size ratio defined as the ratio of grain size to particle size ($a/\beta$). Of this number, there are, on the average, $(k-1)^3$ particles which were contained entirely within a single grain. The remaining particles encompass grain boundaries and are made up of the fragments of a number of grains. The former particles necessarily contain only one mineral species and are termed 'liberated.' Depending on the surrounding grains being of the same or different mineral species, the latter particles may contain only one mineral species or they may contain both mineral species in which case they are termed 'locked.'

It is possible to obtain the number of liberated and locked particles resulting from any one grain by con-

**Table I. List of Nomenclature**

$a$ = a constant
$C$ = volumetric concentration of mineral $B$ in the concentrate, volume of $B$ per unit volume of concentrate
$f$ = frequency function
$k$ = size ratio, the ratio of mineral grain size to particle size
$L$ = degree of liberation, the fraction of a mineral species liberated
$n$ = volumetric abundance ratio, the volume ratio of mineral $A$ to mineral $B$ in the aggregate
$P$ = probability of occurrence
$R$ = recovery of mineral $B$, the fraction, by volume or weight, of $B$ in the feed which is recovered in the concentrate
$s$ = volume fraction of mineral $B$ within a locked particle
$s' = s/k^3$
$t$ = largest integer contained in $1/k$
$u, w, x, y, z$ = distance variables
$V$ = fraction by volume of feed in the concentrate
$v$ = volume fraction of mineral $B$ within a locked particle at point of separation
$v' = v/k^3$
$v_{max}$ = maximum volume fraction of mineral $B$ within a locked particle
$\alpha$ = mineral grain size, linear dimension of a cube
$\beta$ = particle size
$\epsilon$ = fractional remainder defined by Eq. 7

**Subscripts**

$A$ = mineral $A$, the unwanted mineral
$B$ = mineral $B$, the wanted mineral
$AB$ = locked particles containing both minerals $A$ and $B$

sidering the various possible arrangements of the two mineral species in the seven adjacent grains in a unit of $2\times2\times2$ grain cube. On the assumption that the grains of $A$ and $B$ are randomly arranged, the frequency of each possible arrangement or permutation is given by the binomial probability law. Defining the volumetric abundance ratio ($n$) as the ratio of the volume of $A$ to that of $B$ in the aggregate, the probability of each possible arrangement can be expressed as a function of $n$.

By a straightforward counting procedure, the number of each type of particle was obtained for every possible permutation of the eight grain positions. These numbers were combined with the appropriate probability for each permutation and summed for each type of particle to give the following equations:

$$P_A = \frac{(k-1)^3}{k^3}\left(\frac{n}{n+1}\right) + \frac{3(k-1)^2}{k^3}\left(\frac{n}{n+1}\right)^2 + \frac{3(k-1)}{k^3}\left(\frac{n}{n+1}\right)^4 + \frac{1}{k^3}\left(\frac{n}{n+1}\right)^8 \quad [1]$$

$$P_B = \frac{(k-1)^3}{k^3}\left(\frac{1}{n+1}\right) + \frac{3(k-1)^2}{k^3}\left(\frac{1}{n+1}\right)^2 + \frac{3(k-1)}{k^3}\left(\frac{1}{n+1}\right)^4 + \frac{1}{k^3}\left(\frac{1}{n+1}\right)^8 \quad [2]$$

where $P_A$ is the probability of occurrence of a liberated particle of $A$, and $P_B$ that of a liberated particle of $B$. The terms in each equation represent, in order, the probability of a liberated particle being composed of the fragments of one, two, four, and eight different grains of the same mineral species, respectively.

The probability of occurrence of a locked particle containing both $A$ and $B$ is given by

$$\begin{aligned} P_{AB} &= 1 - (P_A + P_B) \\ &= \frac{3(k-1)^2}{k^3}\left[\frac{(n+1)^2 - (n^2+1)}{(n+1)^2}\right] \\ &\quad + \frac{3(k-1)}{k^3}\left[\frac{(n+1)^4 - (n^4+1)}{(n+1)^4}\right] \\ &\quad + \frac{1}{k^3}\left[\frac{(n+1)^8 - (n^8+1)}{(n+1)^8}\right]. \end{aligned} \quad [3]$$

There is no restriction in these equations concerning the value of $n$. As $n$ becomes large, the probability of occurrence of a liberated particle of $A$ increases; as $n$ becomes small, the probability of a liberated particle of $B$ increases. These equations are also applicable to the case of $k$ equal to unity.

**Size Ratio Less Than Unity:** When the particle size ($\beta$) is greater than the grain size ($\alpha$) ($k<1$), a liberated particle must be made up of grains of the same mineral species. Combination of the probability ($\epsilon$ terms) for a given number ($t$ terms) of grains being contained in a particle with the binomial probability ($n$ terms) for all grains being of the same mineral species leads to

$$\begin{aligned} P_A &= (1-\epsilon)^3\left(\frac{n}{n+1}\right)^{(t+1)^3} \\ &\quad + 3\epsilon(1-\epsilon)^2\left(\frac{n}{n+1}\right)^{(t+2)(t+1)^2} \\ &\quad + 3\epsilon^2(1-\epsilon)\left(\frac{n}{n+1}\right)^{(t+2)^2(t+1)} \\ &\quad + \epsilon^3\left(\frac{n}{n+1}\right)^{(t+2)^3} \end{aligned} \quad [4]$$

$$P_B = (1-\epsilon)^3 \left(\frac{1}{n+1}\right)^{(t+1)^3} + 3\epsilon(1-\epsilon)^2 \left(\frac{1}{n+1}\right)^{(t+2)(t+1)^2} + 3\epsilon^2(1-\epsilon) \left(\frac{1}{n+1}\right)^{(t+2)^2(t+1)} + \epsilon^3 \left(\frac{1}{n+1}\right)^{(t+2)^3} \quad [5]$$

$$P_{AB} = 1 - (P_A + P_B) \quad [6]$$

where $t$ is defined as the largest integer contained in $1/k$ and $\epsilon$ as the fractional remainder so that

$$\frac{1}{k} = t + \epsilon. \quad [7]$$

The terms in Eqs. 4 and 5 represent, in order, the probability of having a liberated particle made up of the fragments of $(t+1)$ grains in each of the three dimensions, $(t+1)$ in two dimensions and $(t+2)$ in the third, $(t+1)$ in one dimension and $(t+2)$ in each of the other two, and $(t+2)$ in each of the three dimensions. (Eqs. 1, 2, 4, and 5 are derived in Ref. 2.)

It can be shown that Eqs. 4 and 5 reduce to Eqs. 1 and 2 if $1/k$ is less than unity $(k>1)$ in which case, $t$ is zero and $\epsilon$ is equal to $1/k$. In this sense, Eqs. 4, 5, 6 can be regarded as general expressions for all values of $k$ and $n$.

**Degree of Liberation:** In Fig. 3, the degree of liberation of $B$ is shown as a function of the size ratio $(k)$ and the volumetric abundance ratio $(n)$. The degree of liberation as used by Gaudin[1] is defined by

$$L_A = \frac{n+1}{n} P_A \quad [8]$$

$$L_B = (n+1) P_B. \quad [9]$$

It indicates the fraction of the mineral species which is liberated.

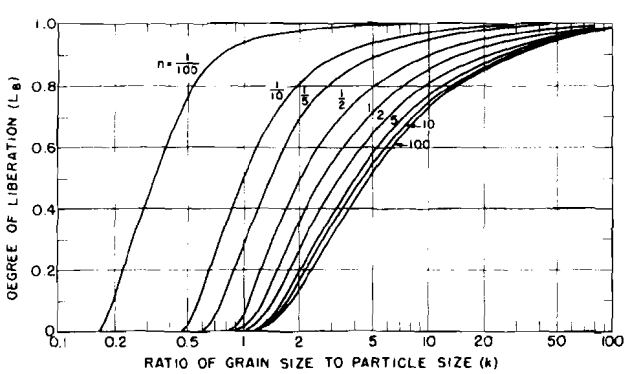


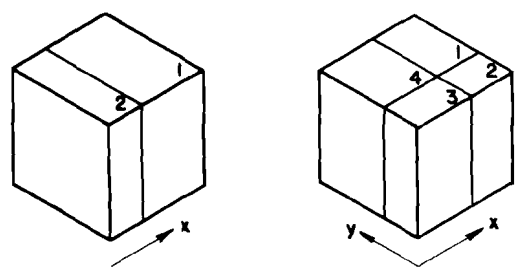

*Fig. 3 — Fraction of mineral B which is liberated as a function of volumetric abundance ratio (n) and size ratio (k).*

Assuming $B$ to be the wanted mineral, it is seen from Fig. 3 that for a practical range of $n$ greater than 1/10, $k$ must be greater than unity in order for significant liberation of $B$ to occur. Hence, from the standpoint of application, the liberation phenomenon for the case of $k$ greater than unity is of great importance. Emphasis will be placed on this case in the following discussions.

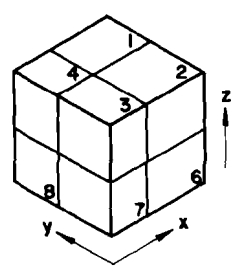
**Mineral Concentration Within Locked Particles:** In order to define the liberation characteristics of the mineral system it is necessary to know not only the amount of either mineral species which is liberated but also the distribution of mineral concentration within the locked fraction.

As illustrated in Fig. 4, there are three distinct types of locked particles resulting from fracture when $k$ is greater than unity. The three types are: those formed at surface boundaries which are composed of the fragments of two dissimilar grains, those formed at edge boundaries which are composed of the fragments of four grains, and those formed at a grain corner, which are composed of the fragments of all eight grains. The probabilities of occurrence of the two-, four-, and eight-fragment particles are given by the first, second, and third terms, respectively, in Eq. 3.

There are, however, numerous combinations and permutations within these three types of locked particles depending on the geometrical arrangement of the two mineral species. The probability of occurrence of each permutation is governed by the binomial probability law. All of the permutations can be grouped into geometrically similar sub-types of the




(a) TWO-GRAIN FRAGMENT LOCKED PARTICLE (b) FOUR-GRAIN FRAGMENT LOCKED PARTICLE




(c) EIGHT-GRAIN FRAGMENT LOCKED PARTICLE

*Fig. 4 — Three distinct types of locked particles.*

two-, four-, and eight-fragment particles. There are a total of 25 such sub-types distributed as follows: one sub-type for the two-fragment particles, four for the four-fragment particles, and 20 for the eight-fragment particles. A list of all the sub-types with the probability of occurrence for each sub-type ($P_i$) is given in Table II where

$$P_{AB} = \sum_{1}^{25} P_i. \qquad [10]$$

Furthermore, each permutation represents a wide range of mineral concentration (volume fraction of a mineral species within each particle) depending on the relative size of the grain fragments contained in a particle. The problem of fragment size can be approached by considering the fracture lattice which was superimposed on the aggregate parallel to the grain lattice, but whose position is random with respect to the grain lattice. This implies that the location of a grain boundary surface within a locked particle is equally likely anywhere along the depth ($x$), width ($y$) and height ($z$) of the particle (Fig. 4). The frequency function for each of the three independent distance variables is therefore equal to unity. By variable transformation,[3] the frequency functions of the distance variables [$f(x)$, $f(y)$, $f(z)$] can be converted into a frequency function for the volume fraction of a mineral species ($f(s_i)$) by the relationship

$$f(s_i) = \int_w \int_u f(x)\, f(y)\, f(z) \left| J \begin{matrix} x, y, z \\ s, u, w \end{matrix} \right|_i du\, dw \qquad [11]$$

for each distinct type or sub-type ($i$) of locked particles, where $f(x) = f(y) = f(z) = 1$; $s_i$ = volume

Table II. Probability of Distinct Locked Particle Sub-types

| Sub-type | Fragments: Total | Fragments: Mineral B | Typical Arrangement of Mineral B Grains* | Permutations in Sub-type | Probability, Pi |
|---|---|---|---|---|---|
| 1 | 2 | 1 | (2) | 2 | $\frac{3(k-1)^2}{k^3}\,\frac{2n}{(n+1)^2}$ |
| 2 | 4 | 1 | (3) | 4 | $\frac{3(k-1)}{k^3}\,\frac{4n^3}{(n+1)^4}$ |
| 3 | 4 | 2 | (2,3) | 4 | $\frac{3(k-1)}{k^3}\,\frac{4n^2}{(n+1)^4}$ |
| 4 | 4 | 2 | (1,3) | 2 | $\frac{3(k-1)}{k^3}\,\frac{2n^2}{(n+1)^4}$ |
| 5 | 4 | 3 | (1,2,3) | 4 | $\frac{3(k-1)}{k^3}\,\frac{4n}{(n+1)^4}$ |
| 6 | 8 | 1 | (7) | 8 | $\frac{1}{k^3}\,\frac{8n^7}{(n+1)^8}$ |
| 7 | 8 | 2 | (5,7) | 12 | $\frac{1}{k^3}\,\frac{12n^6}{(n+1)^8}$ |
| 8 | 8 | 2 | (1,7) | 4 | $\frac{1}{k^3}\,\frac{4n^6}{(n+1)^8}$ |
| 9 | 8 | 2 | (3,7) | 12 | $\frac{1}{k^3}\,\frac{12n^6}{(n+1)^8}$ |
| 10 | 8 | 3 | (1,7,8) | 24 | $\frac{1}{k^3}\,\frac{24n^5}{(n+1)^8}$ |
| 11 | 8 | 3 | (2,4,7) | 8 | $\frac{1}{k^3}\,\frac{8n^5}{(n+1)^8}$ |
| 12 | 8 | 3 | (3,7,8) | 24 | $\frac{1}{k^3}\,\frac{24n^5}{(n+1)^8}$ |
| 13 | 8 | 4 | (1,3,7,8) | 24 | $\frac{1}{k^3}\,\frac{24n^4}{(n+1)^8}$ |
| 14 | 8 | 4 | (1,2,3,7) | 24 | $\frac{1}{k^3}\,\frac{24n^4}{(n+1)^8}$ |

(Continued next page)

Table II. Probability of Distinct Locked Particle Sub-types (Continued)

| Sub-type | Fragments: Total | Fragments: Mineral B | Typical Arrangement of Mineral B Grains* | Permutations in Sub-type | Probability, Pi |
|---|---|---|---|---|---|
| 15 | 8 | 4 | (1,2,7,8) | 6 | $\frac{1}{k^3}\frac{6n^4}{(n+1)^8}$ |
| 16 | 8 | 4 | (3,6,7,8) | 8 | $\frac{1}{k^3}\frac{8n^4}{(n+1)^8}$ |
| 17 | 8 | 4 | (2,4,5,7) | 2 | $\frac{1}{k^3}\frac{2n^4}{(n+1)^8}$ |
| 18 | 8 | 4 | (5,6,7,8) | 6 | $\frac{1}{k^3}\frac{6n^4}{(n+1)^8}$ |
| 19 | 8 | 5 | (1,2,5,6,7) | 24 | $\frac{1}{k^3}\frac{24n^3}{(n+1)^8}$ |
| 20 | 8 | 5 | (2,3,4,5,7) | 8 | $\frac{1}{k^3}\frac{8n^3}{(n+1)^8}$ |
| 21 | 8 | 5 | (1,3,4,6,7) | 24 | $\frac{1}{k^3}\frac{24n^3}{(n+1)^8}$ |
| 22 | 8 | 6 | (1,2,3,5,6,7) | 12 | $\frac{1}{k^3}\frac{12n^2}{(n+1)^8}$ |
| 23 | 8 | 6 | (1,3,4,5,6,7) | 4 | $\frac{1}{k^3}\frac{4n^2}{(n+1)^8}$ |
| 24 | 8 | 6 | (1,2,3,4,5,7) | 12 | $\frac{1}{k^3}\frac{12n^2}{(n+1)^8}$ |
| 25 | 8 | 7 | (1,2,4,5,6,7,8) | 8 | $\frac{1}{k^3}\frac{8n}{(n+1)^8}$ |

*Arrangement refers to Fig. 4 a, b, or c for 2, 4, or 8 fragment locked particles, respectively.

fraction of a mineral species in the $i$-th sub-type of locked particles $= s_i(x, y, z)$; $u = y$; and $w = z$.

The volume fraction frequency function for all locked particles $[f(s)]$ is obtained by

$$f(s) = \frac{\sum_1^{25} P_i f(s_i)}{P_{AB}} \qquad [12]$$

and the fraction of locked particles containing more than a specified volume fraction ($v$) of the wanted mineral which is designated to be mineral $B$ is given by

$$\int_v^1 f(s)\, ds = \frac{\sum_1^{25} P_i \int_v^1 f(s_i)\, ds}{P_{AB}} . \qquad [13]$$

The amount (in volume) of the wanted mineral in that fraction is then

$$\int_v^1 s\, f(s)\, ds = \frac{\sum_1^{25} P_i \int_v^1 s\, f(s_i)\, ds}{P_{AB}} . \qquad [14]$$

Numerical values for the integrals $\int_v^1 f(s_i)\, ds$ in Eq. 13 and $\int_v^1 s\, f(s_i)\, ds$ in Eq. 14 at uniform intervals of $v$ have been calculated for each distinct type of locked particle and are tabulated in Tables III and IV.

**Concentration - Recovery Diagram:** A criterion frequently used in evaluating the liberation characteristics of a mineral system is the concentration-recovery diagram or the grade-recovery diagram. In this diagram, the fraction of the wanted mineral recovered in the concentrate is plotted against the concentration of the wanted mineral in the concentrate. Data for this diagram is obtained using a separation process which is selective toward one of the mineral species. Heavy liquid separation is one such process commonly used, which is based on differences in specific gravity of the mineral species. The point of separation can be changed by adjusting an operating variable in the separation process, for example, the liquid density in the heavy liquid process, which results in changes in the recovery

Table III. Numerical Values of $\int_v^1 f(s_i)\,ds$ for Distinct Locked Particle Sub-types

| Sub-type | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.1000 | 0.0052 | 0.1000 | 0.0107 | 0.3303 |
| 0.8 | 0.2000 | 0.0215 | 0.2000 | 0.0468 | 0.5219 |
| 0.7 | 0.3000 | 0.0503 | 0.3000 | 0.1167 | 0.6612 |
| 0.6 | 0.4000 | 0.0935 | 0.4000 | 0.2391 | 0.7665 |
| 0.5 | 0.5000 | 0.1534 | 0.5000 | 0.5000 | 0.8466 |
| 0.4 | 0.6000 | 0.2335 | 0.6000 | 0.7609 | 0.9065 |
| 0.3 | 0.7000 | 0.3388 | 0.7000 | 0.8833 | 0.9497 |
| 0.2 | 0.8000 | 0.4781 | 0.8000 | 0.9532 | 0.9785 |
| 0.1 | 0.9000 | 0.6697 | 0.9000 | 0.9893 | 0.9948 |
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| **Sub-type** | **6** | **7*** | **8*** | **9** | **10*** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.0002 | $0.000^+$ | $0.000^+$ | 0.0052 | 0.010 |
| 0.8 | 0.0016 | 0.002 | 0.002 | 0.0215 | 0.031 |
| 0.7 | 0.0058 | 0.010 | 0.008 | 0.0503 | 0.065 |
| 0.6 | 0.0152 | 0.038 | 0.034 | 0.0935 | 0.125 |
| 0.5 | 0.0333 | 0.088 | 0.064 | 0.1534 | 0.236 |
| 0.4 | 0.0656 | 0.206 | 0.138 | 0.2335 | 0.424 |
| 0.3 | 0.1214 | 0.358 | 0.272 | 0.3388 | 0.632 |
| 0.2 | 0.2191 | 0.558 | 0.586 | 0.4781 | 0.810 |
| 0.1 | 0.4046 | 0.762 | 0.886 | 0.6697 | 0.941 |
| 0 | 1.0000 | 1.000 | 1.000 | 1.0000 | 1.000 |
| **Sub-type** | **11*** | **12*** | **13*** | **14** | **15** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | $0.000^+$ | 0.024 | 0.024 | 0.0377 | 0.0107 |
| 0.8 | 0.003 | 0.070 | 0.076 | 0.1216 | 0.0468 |
| 0.7 | 0.021 | 0.136 | 0.159 | 0.2336 | 0.1167 |
| 0.6 | 0.069 | 0.218 | 0.286 | 0.3627 | 0.2391 |
| 0.5 | 0.189 | 0.317 | 0.500 | 0.5000 | 0.5000 |
| 0.4 | 0.510 | 0.430 | 0.714 | 0.6373 | 0.7609 |
| 0.3 | 0.698 | 0.560 | 0.841 | 0.7664 | 0.8833 |
| 0.2 | 0.837 | 0.699 | 0.924 | 0.8784 | 0.9532 |
| 0.1 | 0.944 | 0.848 | 0.976 | 0.9623 | 0.9893 |
| 0 | 1.000 | 1.000 | 1.000 | 1.0000 | 1.0000 |
| **Sub-type** | **16*** | **17** | **18** | **19*** | **20*** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.056 | 0.0008 | 0.1000 | 0.152 | 0.056 |
| 0.8 | 0.153 | 0.0076 | 0.2000 | 0.301 | 0.163 |
| 0.7 | 0.256 | 0.0328 | 0.3000 | 0.440 | 0.302 |
| 0.6 | 0.373 | 0.1096 | 0.4000 | 0.570 | 0.490 |
| 0.5 | 0.500 | 0.5000 | 0.5000 | 0.683 | 0.811 |
| 0.4 | 0.627 | 0.8904 | 0.6000 | 0.782 | 0.931 |
| 0.3 | 0.744 | 0.9672 | 0.7000 | 0.864 | 0.979 |
| 0.2 | 0.847 | 0.9924 | 0.8000 | 0.930 | 0.997 |
| 0.1 | 0.944 | 0.9992 | 0.9000 | 0.976 | $1.000^-$ |
| 0 | 1.000 | 1.0000 | 1.0000 | 1.000 | 1.000 |
| **Sub-type** | **21*** | **22** | **23*** | **24*** | **25** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.059 | 0.3303 | 0.114 | 0.238 | 0.5954 |
| 0.8 | 0.190 | 0.5219 | 0.414 | 0.442 | 0.7809 |
| 0.7 | 0.368 | 0.6612 | 0.728 | 0.642 | 0.8785 |
| 0.6 | 0.576 | 0.7665 | 0.862 | 0.794 | 0.9344 |
| 0.5 | 0.764 | 0.8466 | 0.936 | 0.912 | 0.9667 |
| 0.4 | 0.875 | 0.9065 | 0.966 | 0.962 | 0.9848 |
| 0.3 | 0.935 | 0.9497 | 0.992 | 0.990 | 0.9942 |
| 0.2 | 0.969 | 0.9785 | 0.998 | 0.998 | 0.9984 |
| 0.1 | 0.990 | 0.9948 | $1.000^-$ | $1.000^-$ | 0.9998 |
| 0 | 1.000 | 1.0000 | 1.000 | 1.000 | 1.0000 |

*Due to difficulties encountered in obtaining definite integrals, these data represent numerical approximations to their values.

Table IV. Numerical Values of $\int_v^1 sf(s_i)\,ds$ for Distinct Locked Particle Sub-types

| Sub-type | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.0950 | 0.0049 | 0.0950 | 0.0101 | 0.3163 |
| 0.8 | 0.1800 | 0.0186 | 0.1800 | 0.0404 | 0.4797 |
| 0.7 | 0.2550 | 0.0401 | 0.2550 | 0.0926 | 0.5845 |
| 0.6 | 0.3200 | 0.0680 | 0.3200 | 0.1715 | 0.6532 |
| 0.5 | 0.3750 | 0.1009 | 0.3750 | 0.3125 | 0.6975 |
| 0.4 | 0.4200 | 0.1367 | 0.4200 | 0.4325 | 0.7246 |
| 0.3 | 0.4550 | 0.1733 | 0.4550 | 0.4758 | 0.7398 |
| 0.2 | 0.4800 | 0.2078 | 0.4800 | 0.4936 | 0.7471 |
| 0.1 | 0.4950 | 0.2360 | 0.4950 | 0.4993 | 0.7497 |
| 0 | 0.5000 | 0.2500 | 0.5000 | 0.5000 | 0.7500 |
| **Sub-type** | **6** | **7*** | **8*** | **9** | **10*** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.0002 | $0.000^+$ | $0.000^+$ | 0.0049 | 0.009 |
| 0.8 | 0.0014 | 0.002 | 0.002 | 0.0186 | 0.026 |
| 0.7 | 0.0045 | 0.008 | 0.006 | 0.0401 | 0.051 |
| 0.6 | 0.0105 | 0.026 | 0.023 | 0.0680 | 0.089 |
| 0.5 | 0.0204 | 0.053 | 0.039 | 0.1009 | 0.149 |
| 0.4 | 0.0348 | 0.105 | 0.072 | 0.1367 | 0.233 |
| 0.3 | 0.0540 | 0.158 | 0.119 | 0.1733 | 0.306 |
| 0.2 | 0.0780 | 0.208 | 0.196 | 0.2078 | 0.351 |
| 0.1 | 0.1047 | 0.238 | 0.243 | 0.2360 | 0.371 |
| 0 | 0.1250 | 0.250 | 0.250 | 0.2500 | 0.375 |
| **Sub-type** | **11*** | **12*** | **13*** | **14** | **15** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | $0.000^+$ | 0.022 | 0.022 | 0.0353 | 0.0101 |
| 0.8 | 0.003 | 0.060 | 0.066 | 0.1063 | 0.0404 |
| 0.7 | 0.016 | 0.109 | 0.127 | 0.1901 | 0.0926 |
| 0.6 | 0.047 | 0.162 | 0.209 | 0.2739 | 0.1715 |
| 0.5 | 0.111 | 0.216 | 0.324 | 0.3494 | 0.3125 |
| 0.4 | 0.256 | 0.267 | 0.423 | 0.4112 | 0.4325 |
| 0.3 | 0.321 | 0.312 | 0.468 | 0.4565 | 0.4758 |
| 0.2 | 0.356 | 0.346 | 0.490 | 0.4847 | 0.4936 |
| 0.1 | 0.372 | 0.368 | 0.498 | 0.4976 | 0.4993 |
| 0 | 0.375 | 0.375 | 0.500 | 0.5000 | 0.5000 |
| **Sub-type** | **16*** | **17** | **18** | **19*** | **20*** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.052 | 0.0007 | 0.0950 | 0.145 | 0.053 |
| 0.8 | 0.135 | 0.0065 | 0.1800 | 0.272 | 0.144 |
| 0.7 | 0.212 | 0.0251 | 0.2550 | 0.377 | 0.248 |
| 0.6 | 0.288 | 0.0743 | 0.3200 | 0.462 | 0.371 |
| 0.5 | 0.358 | 0.2812 | 0.3750 | 0.524 | 0.547 |
| 0.4 | 0.415 | 0.4647 | 0.4200 | 0.569 | 0.603 |
| 0.3 | 0.456 | 0.4923 | 0.4550 | 0.598 | 0.620 |
| 0.2 | 0.482 | 0.4988 | 0.4800 | 0.616 | $0.625^-$ |
| 0.1 | 0.496 | 0.4999 | 0.4950 | 0.623 | $0.625^-$ |
| 0 | 0.500 | 0.5000 | 0.5000 | 0.625 | 0.625 |
| **Sub-type** | **21*** | **22** | **23*** | **24*** | **25** |
| $v = 1.0$ | 0 | 0 | 0 | 0 | 0 |
| 0.9 | 0.055 | 0.3163 | 0.107 | 0.226 | 0.5751 |
| 0.8 | 0.166 | 0.4797 | 0.360 | 0.400 | 0.7339 |
| 0.7 | 0.299 | 0.5845 | 0.597 | 0.550 | 0.8075 |
| 0.6 | 0.434 | 0.6532 | 0.684 | 0.649 | 0.8441 |
| 0.5 | 0.538 | 0.6975 | 0.725 | 0.715 | 0.8621 |
| 0.4 | 0.589 | 0.7246 | 0.739 | 0.738 | 0.8704 |
| 0.3 | 0.611 | 0.7398 | 0.748 | 0.748 | 0.8737 |
| 0.2 | 0.620 | 0.7471 | $0.750^-$ | $0.750^-$ | 0.8748 |
| 0.1 | 0.624 | 0.7497 | $0.750^-$ | $0.750^-$ | $0.8750^-$ |
| 0 | 0.625 | 0.7500 | 0.750 | 0.750 | 0.8750 |

*Due to difficulties encountered in obtaining definite integrals, these data represent numerical approximations to their values.

and the concentration of the wanted mineral in the concentrate. These diagrams are usually constructed on a weight basis, but can be readily converted to a volume basis since recovery is the same in either system and volume concentration can be calculated from weight concentration based on the principles of additive volume.

If the separation process is assumed to be ideal, being capable of a clean separation of particles based entirely on the volumetric concentration of each particle, an expression for the concentration-recovery curve can be derived from the probability equations and frequency functions given in the preceding sections.

Consider, for convenience of explanation, the heavy liquid process where separation is effected by using liquids of specific gravity intermediate between those of the mineral species. For a given specific gravity (or liquid) which corresponds to a certain value of particle volume fraction $v$ of the wanted mineral, the concentrate from this separation would consist of all particles with a volume fraction greater than $v$. Assuming again the wanted mineral to be $B$, the fraction by volume of the feed appearing in the concentrate is given by

$$V = P_B + P_{AB} \int_v^1 f(s)\, ds = P_B + \sum_1^{25} P_i \int_v^1 f(s_i)\, ds \qquad [15]$$

and the volumetric concentration of $B$ in the concentrate by

$$C = \frac{P_B + P_{AB} \int_v^1 s\, f(s)\, ds}{V} = \frac{P_B + \sum_1^{25} P_i \int_v^1 s\, f(s_i)\, ds}{P_B + \sum_1^{25} P_i \int_v^1 f(s_i)\, ds}. \qquad [16]$$

Accordingly, the recovery of $B$ defined as the fraction, by volume or weight, of $B$ in the feed which is recovered in the concentrate is

$$R = (n+1)\left[P_B + P_{AB} \int_v^1 s\, f(s)\, ds\right] = (n+1)\left[P_B + \sum_1^{25} P_i \int_v^1 s\, f(s_i)\, ds\right]. \qquad [17]$$

Values of $V$, $C$, and $R$ can now be calculated for specific values of the volumetric abundance ratio ($n$) and the size ratio ($k$) with the particle volume fraction at the point of separation ($v$) as a parameter by using $P_B$ given by Eq. 2, $P_i$ in Table II and the numerical values of the integrals in Tables III and IV. As an illustration, Fig. 5 shows a concentration-recovery diagram for $n$ equal to 4 and $k$ equal to 1, 2, 4, 8, and 16.

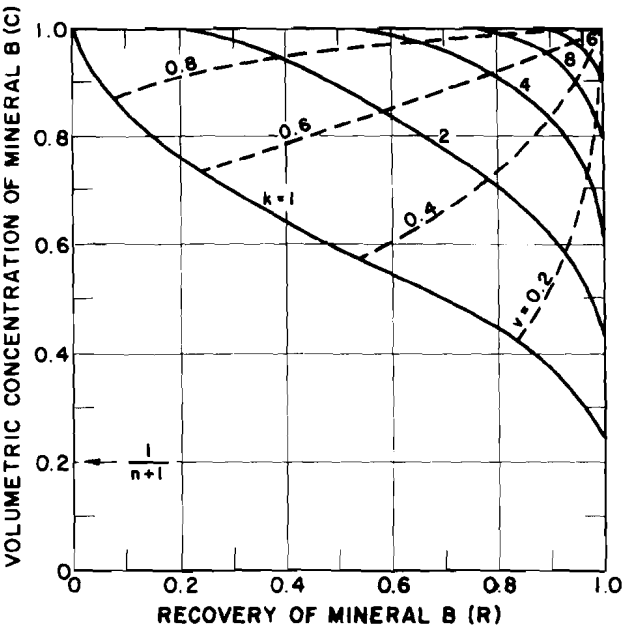



*Fig. 5 — Concentration-recovery diagram for the random model for n = 4.*

## 'SOLITARY-GRAIN' MODEL

For the particular case of a binary mineral system which contains only trace quantities of the wanted mineral, a much simpler model can be used to approximate the process of liberation by size reduction. In this model which will be referred to as the 'solitary-grain' model, the aggregate is assumed to be represented by a solitary cubic grain of the wanted mineral completely surrounded by cubic grains of the other mineral. The same assumption is made regarding the fracture of this aggregate as was made for the random model. The equations for this solitary-grain model can be derived from the physical model itself or from the equations for the random model by allowing the abundance ratio ($n$) to become large.

Consider first the case of the size ratio ($k$) greater than unity, the number of liberated particles of the wanted mineral is $(k-1)^3$. The locked particles are distributed as follows: six $(k-1)^2$ of the two-fragment type, twelve $(k-1)$ of the four-fragment type, and eight of the eight-fragment type. If $B$ is the wanted mineral, the probabilities of occurrence of liberated and locked particles are given by

$$P_A = 1 - \frac{(k+1)^3}{k^3}\left(\frac{1}{n+1}\right) \qquad [18]$$

$$P_B = \frac{(k-1)^3}{k^3}\left(\frac{1}{n+1}\right) \qquad [19]$$

$$P_{AB} = 1 - (P_A + P_B) = \left[\frac{6(k-1)^2}{k^3} + \frac{12(k-1)}{k^3} + \frac{8}{k^3}\right]\left(\frac{1}{n+1}\right). \qquad [20]$$

It is to be noted that Eq. 18 can also be obtained from Eq. 1 by making the following approximation for large values of $n$

$$\left(\frac{n}{n+1}\right)^a = \left(1 - \frac{1}{n+1}\right)^a \cong 1 - \frac{a}{n+1}. \qquad [21]$$

The volume fraction frequency functions ($f(s_i)$) are 1, $-ln\ s$, and $(ln\ s)^2/2$ for two- four- and eight-fragment locked particles, respectively. These functions combined with their probabilities lead to the following analytical expressions for the overall frequency function and related functions

$$f(s) = \frac{6(k-1)^2 - 12(k-1)\,ln\ s + 4(ln\ s)^2}{(k+1)^3 - (k-1)^3} \qquad [22]$$

particle. Therefore, the probability of occurrence of locked particles is

$$P_{AB} = \frac{8k^3 + 12k^2(1-k) + 6k(1-k)^2 + (1-k)^3}{(n+1)k^3} = \frac{(k+1)^3}{k^3}\left(\frac{1}{n+1}\right). \qquad [28]$$

The fourth type of locked particles which contains an entire grain of $B$ has the maximum volume fraction of $B$ given by

$$v_{max} = \frac{\alpha^3}{\beta^3} = k^3 \qquad [29]$$

$$\int_v^1 f(s)\,ds = \frac{(k+1)^3 - (k-1)^3 - [(k+1)^3 - (k-1)^3]\,v + 4[3(k-1)+2]\,v\,ln\ v - 4v(ln\ v)^2}{(k+1)^3 - (k-1)^3} \qquad [23]$$

$$\int_v^1 s\,f(s)\,ds = \frac{k^3 - (k-1)^3 - [k^3 - (k-1)^3]\,v^2 + 2[3(k-1)+1]\,v^2 ln\ v - 2v^2(ln\ v)^2}{(k+1)^3 - (k-1)^3}. \qquad [24]$$

Accordingly, $V$, $C$, and $R$ as defined by Eqs. 15, 16, and 17 are given by

$$V = \frac{(k+1)^3 - [(k+1)^3 - (k-1)^3]\,v + 4[3(k-1)+2]\,v\,ln\ v - 4v(ln\ v)^2}{k^3\,(n+1)} \qquad [25]$$

$$C = \frac{k^3 - [k^3 - (k-1)^3]\,v^2 + 2[3(k-1)+1]\,v^2 ln\ v - 2v^2(ln\ v)^2}{(k+1)^3 - [(k+1)^3 - (k-1)^3]\,v + 4[3(k-1)+2]\,v\,ln\ v - 4v(ln\ v)^2} \qquad [26]$$

$$R = \frac{k^3 - [k^3 - (k-1)^3]\,v^2 + 2[3(k-1)+1]\,v^2 ln\ v - 2v^2(ln\ v)^2}{k^3}. \qquad [27]$$

When the size ratio is less than unity, there would be no liberated particles of $B$, the wanted mineral, and four types of locked particles are possible depending on the position of the fracture lattice relative to the solitary grain. Consider one point in the fracture lattice to be the intersection of three planes. The probability of the solitary grain of size $\alpha$ being cut by all three planes is $k^3$ which yields eight locked particles; the probability of it being cut by two is $3\,k^3\,(1-k)$ which yields four locked particles; the probability of it being cut by one is $3\,k\,(1-k)^2$ which yields two locked particles, and finally the probability of it not being cut by any of the three planes is $(1-k)^3$ which yields only one locked

and the probability of its occurrence is

$$P_{AB}(v_{max}) = \frac{(1-k)^3}{k^3}\left(\frac{1}{n+1}\right). \qquad [30]$$

For the first three types of locked particles, the overall frequency function and related functions can be obtained analytically as

$$f(s) = \frac{6k(1-k)^2 - 12k^2(1-k)\,ln(s') + 4k^3[ln(s')]^2}{k^3[(1+k)^3 - (1-k)^3]} \qquad [31]$$

$$\int_v^{k^3} f(s)\,ds = \frac{1}{(1+k)^3 - (1-k)^3}\{(1+k)^3 - (1-k)^3 - [(1+k)^3 - (1-k)^3]\,(v') + 4[3k^2(1-k) + 2k^3]\,(v')ln(v') - 4k^3(v')[ln(v')]^2\} \qquad [32]$$

$$\int_v^{k^3} s\,f(s)ds = \frac{k^3}{(1+k)^3 - (1-k)^3}\{1 - (1-k)^3 - [1-(1-k)^3]\,(v')^2 + 2[3k^2(1-k) + k^3](v')^2\,ln(v') - 2k^3(v')^2[ln(v')]^2\} \qquad [33]$$

where $s' = (s/k^3)$, $0 \le s' \le 1$ and $v' = (v/k^3)$, $0 \le v' \le 1$.

The fraction by volume of the feed appearing in the concentrate for this case becomes

$$V = P_{AB}(v_{max}) + [P_{AB} - P_{AB}(v_{max})] \int_v^{k^3} f(s)\,ds \qquad [34]$$

and the concentration-recovery curve is then defined by

$$C = \frac{P_{AB}(v_{max})\,k^3 + [P_{AB} - P_{AB}(v_{max})] \int_v^{k^3} s\,f(s)\,ds}{P_{AB}(v_{max}) + [P_{AB} - P_{AB}(v_{max})] \int_v^{k^3} f(s)\,ds} \qquad [35]$$

$$R = (n+1)\{P_{AB}(v_{max})\,k^3 + [P_{AB} - P_{AB}(v_{max})] \int_v^{k^3} s\,f(s)\,ds\}. \qquad [36]$$

Substitution of Eq. 28, 30, 31, 32, and 33 into Eqs. 34, 35, and 36 gives

$$V = \frac{1}{k^3(n+1)}\{(1+k)^3 - [(1+k)^3 - (1-k)^3](v') + 4[3k^2(1-k) + 2k^3](v')\,ln\,(v') - 4k^3(v')[ln\,(v')]^2\} \qquad [37]$$

$$C = \frac{k^3\{1 - [1-(1-k)^3]v'^2 + 2[3k^2(1-k) + k^3]v'^2\,lnv' - 2k^3v'^2[lnv']^2\}}{(1+k)^3 - [(1+k)^3 - (1-k)^3]v' + 4[3k^2(1-k) + 2k^3]v'ln\,v' - 4k^3v'[lnv']^2} \qquad [38]$$

$$R = 1 - [1-(1-k)^3]v'^2 + 2[3k^2(1-k) + k^3]v'^2\,ln\,v' - 2k^3v'^2[lnv']^2. \qquad [39]$$

Eqs. 26 and 27 as well as Eqs 38 and 39 indicate that for the solitary grain model the concentration-recovery relationship is independent of the abundance ratio ($n$). Fig. 6 shows a concentration-recovery diagram for various values of $k$, both less and greater than unity. This diagram clearly demonstrates the impossibility of obtaining a high concentration of the wanted mineral if the size ratio is less than unity, and also the necessity of having a size ratio greater than four to obtain a high concentration at a reasonable recovery.

The value of the solitary-grain model lies not only in its simulation of a mineral system containing trace quantities of wanted mineral, but also in its definition of the lower limit of the concentration-recovery curve obtainable from the random model. That is, regardless of the value of the volumetric abundance ratio, the worst possible results for a particular value of size ratio are described by the solitary-grain model.

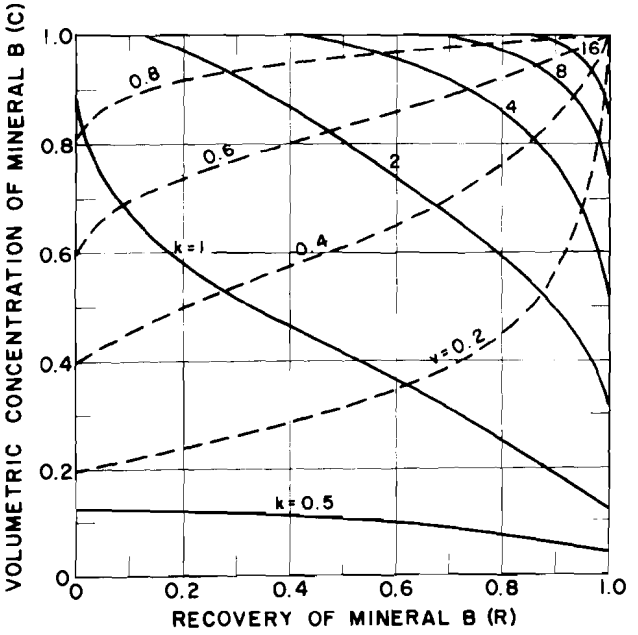



*Fig. 6 — Concentration-recovery diagram for the solitary-grain model.*

## DISCUSSION

The physical models treated here are necessarily based on certain simplifying assumptions regarding the number of mineral species; the size, shape, and arrangement of mineral grains; the mechanism of fracture; and the size and shape of the resulting particles. It is of interest to consider the effect of departures from these assumptions.

Although the theoretical treatment was limited to the case of a binary mineral system, the same technique can be applied to ternary or higher order systems, by making use of multinomial probability functions instead of binomial functions. The problem of relating locked particle concentration to frequency of occurrence, however, becomes considerably more complex. An alternative is to simplify the system to the consideration of one valuable mineral and the remaining gangue minerals.

Real mineral systems display much more complex grain structure than the aligned cubic grain structure assumed here. There are, however, a number of industrially important minerals which approach the cubic shape. The fact that real mineral grains are not aligned corner to corner probably does not have a large effect on the validity of the model. On the other hand, it is more important that the mineral grains be

aligned face to face, which is usually the case for real mineral systems.

The existence of a grain size distribution within the mineral system would result in departures from theoretical predictions. If the distribution is narrow, predictions could be based on the average grain size, or in the case of the solitary-grain model, an integration over the distribution could be made. However, if the distribution is quite wide, as, for example, where the unwanted mineral appears as grains of a size similar to the wanted mineral and also as minute inclusions in the wanted mineral, it may become necessary to treat the system as though the wanted mineral were a mixture of pure wanted mineral and inclusions.

Consideration has been limited to random fractures, whereas in reality some systems exhibit a tendency to fracture at grain boundaries or to preferentially fracture one of the components. Breaking at grain boundaries would result in significantly better liberation than expected. Preferential fracture would lead to one component appearing as smaller particles than the other, which could be detected from an assay-size fraction plot. The effect of this behavior on liberation is debatable.

In reality each fracture results in the production of a size distribution of random-shaped particles as opposed to cubic particles of uniform size. Most mineral separations are also made on a size distribution and not discrete size fractions. Because of the large number of particles produced in a size reduction operation, it is reasonable to assume that each specific size fraction does represent the entire spectrum of particles which would be obtained if the fracture had resulted in the production of only that specific uniform size. Prediction of the concentration-recovery relationship for a size distribution could be based on an integration or summation over the distribution, if sufficient information concerning the distribution and the separation process are available. The simplest case where the separation is based on mineral concentration alone, such as heavy liquid, may be handled by direct summation or integration with respect to particle size

$$V = \int_{\beta} g(\beta)\, V_{\beta}\, d_{\beta} \qquad [40]$$

$$R = \int_{\beta} g(\beta)\, R_{\beta}\, d_{\beta} \qquad [41]$$

$$C = \frac{1}{(n+1)}\,\frac{R}{V} \qquad [42]$$

where $g(\beta)$ is the particle size frequency function, and $V_{\beta}$ and $R_{\beta}$ are the values of $V$ and $R$ for a given particle size $\beta$, respectively. But for more complicated separations, where there is a relationship between mineral concentration at the point of separation and particle size, integration becomes more difficult.

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# DISCUSSION

## ON THE RELATIONSHIP BETWEEN CONTACT ANGLE AND FLOTATION BEHAVIOR


by R. W. Smith and R. W. M. Lai



*Transactions of SME, December 1966, p. 413; AIME Transactions, 1966, vol. 235.*


**J. Leja** *(Professor, Dept. of Mineral Engineering, U.B.C. Vancouver, Canada)* – I would like to offer some comments on the behavior of contact angle with time, treated in this paper. The authors should be congratulated for a clear exposition of these uncommon departures in the correlation between flotation and contact angle, in systems of importance to industrial practice. Some nine years ago Professor J. H. Schulman (presently at Columbia University, New York) and I reported on a similar study conducted at the Ernest Oppenheimer Laboratory, Cambridge, England.[1] The systems used by us had, however, no direct practical application at that time, as mercury in solutions of xanthates was tested with a variety of alkyl-aryl polyoxyethylene frothers which were then new to flotation practice. Further, the paper dealt with findings contrary to the then widely held belief that the contact angle established in collector-solution is unaffected by the addition of a frother.

Correlation with flotation, or lack thereof, was not sought, though we did suggest that in many cases the selectivity of frothers might be influenced by the dynamic contact angles. We also stated (on p. 243,

The optimization techniques, formatting the mathematical model and adequate model for carried out investigations, by means of tables and figures will show the optimal quantity in reagent regime (collectors), particle size, flotation time, rougher flotation, conditioning time etc.Â  Empirical modeling and aptinusation of mineral processes. A. L. Mular. Mineral Science and Engineering, 1980. Gridrocikloni na obogatitelâ€™nyh fabrikahâ€, â€œNedra. A I.Povarov. 1978. Measurement and interpretation of size distribution of particles within a hydrocyclone. V. G. Renner, H. E. Cohen. (June Download Citation on ResearchGate | Size reduction/mineral liberation simulation for a magnetic taconite concentrator | Several mineral process modeling and simulation packages are currently available for use in evaluating the performance of alternative process flow schemes. A critical link that has been missing from these packages is the ability to quantitatively model the effect of size...Â  The results of this integration of size reduction and mineral liberation models are discussed from both theoretical and practical viewpoints using batch grinding data for a magnetic iron formation sample as a demonstration vehicle. View. Show abstract. A random model for mineral liberation by size reduction. Article. Jan 1967.