# SMITH INSTITUTE FARADAY PARTNERSHIP FAST TRACK PROPOSAL
# CASE FOR SUPPORT

## ELECTROMAGNETIC INVERSE PROBLEMS FOR LIQUID CRYSTAL DISPLAYS AND CAPACITANCE IMAGING

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### Part I: Previous Track Record

**Dr William Lionheart** (Principal Investigator) has worked in non-linear inverse problems for 18 years and has over 40 refereed publications in that area. Initially specialising in reconstruction algorithms for electrical impedance imaging he founded an experimental group in that area at Oxford Brookes. Funded by two Wellcome Trust grants his group developed a series of adaptive current tomographs, and participated in two EU concerted actions where Lionheart was an invited speaker at each meeting. The UMIST group plays a central role in the British Inverse Problems community. Lionheart shares with Kurylev at Loughborough the organisation of British Workshops on Inverse Problems. These meetings provide a forum for interaction between those working on abstract questions of uniqueness and stability, numerical algorithms and mathematical modelling as well as experimental scientists and engineers interested in measurement and applications.

On moving to UMIST in 1999, where Lionheart is now a Reader, his work expanded to include industrial imaging problems, both electrical and optical. He leads a growing Inverse Problems Group in the Mathematics Department with funding from and collaboration with industry (e.g. Corus Group, Philips, QinetiQ also several SMEs), as well as collaborative links with other leading academic inverse problems groups around the world. He is currently co-authoring a book on reconstruction algorithms for non-linear inverse problems with Arridge (UCL).

In addition to work on reconstruction algorithms his theoretical work specialises in uniqueness results for anisotropic electromagnetic problems using geometric and microlocal methods [L1,L2,L3]. His student Romina Gaburro, working with Alessandrini in Trieste generalised a uniqueness result of Lionheart [L1] for the anisotropic inverse conductivity problem [G1]. She now works on extensions of this work in the UMIST group. Lionheart collaborates with Kurylev at Loughborough using methods of global analysis to investigate uniqueness and stability issues in anisotropic inverse problems, also Somersalo in Helsinki on a geometric approach to anisotropic Maxwell's equations, and Alessandrini in Trieste on analytical methods for uniqueness and stability of anisotropic inverse problems.

The results of this more abstract mathematical work is fed back to the applications community using the close collaborative relationship Lionheart has with both academic and industrial engineers and scientists. Lionheart often presents invited papers at conferences on Bio-medical Engineering [L4] and on Industrial Process Tomography [L5].

Lionheart coordinates the EIDORS (Electrical Impedance and Diffuse Optical Reconstruction Software) project [L6], a collaboration principally with UMIST, Kuopio (Finland) and UCL to develop open source image reconstruction software for electrical and diffuse optical imaging in medicine and industry.

He holds EPSRC Grant GR/R64278 on electromagnetic imaging of molten steel, a project jointly with the engineering Department at Lancaster, Corus Group at Teeside, and a smaller company MPI. This project involves using edge finite elements for the eddy current equations. Lionheart currently has three PhD students in the Mathematics Department, and co-supervises three more in engineering departments, all working on inverse problems.

**Selected References**

[L1] WRB Lionheart, Conformal uniqueness results in anisotropic electrical impedance imaging. Inv. Probl. **13**, 125–134, (1997)

[L2] WRB Lionheart, Boundary shape and electrical impedance tomography, Inv. Probl. **14**, 139–147, (1998)

[L3] M Joshi, W Lionheart, An inverse boundary value problem for harmonic differential forms. submitted to Asyptotoic Anal., preprint http://www.arxiv.org/abs/math.AP/9911212

[L4] WRB Lionheart. Uniqueness, shape, and dimension in EIT, ANNALS OF THE NEW YORK ACADEMY OF SCIENCES, **873**,466-471,(1999)

[L5] WRB Lionheart, Proceedings of the 2nd World Congress on Industrial Process Tomography, Hannover, Germany, August, Invited key-note on page 4, coauthored papers on pages 272, 387 and 433 (2001).

[L6] M Vauhkonen, WRB Lionheart, LM Heikkinen, PJ Vauhkonen and JP Kaipio, A MATLAB package for the EIDORS project to reconstruct two-dimensional EIT images, Physiol. Meas. **22** 107-111. Other coauthored papers on pages 77-83, 85-90 and 147-157 (2001).

[G1] G Alessandrini, R Gaburro., Determining conductivity with special anisotropy by boundary measurements. SIAM J. Math. Anal. **33**, (2001) 153-171

**Dr Nigel Mottram** (Co-Investigator) is a Lecturer in the Liquid Crystal Theory group and an EPSRC Advanced Research Fellow in the Department of Mathematics at the University of Strathclyde. He has been involved in research into the theory of liquid crystal materials since 1992 when he began his Ph.D. under the supervision of Prof S.J. Hogan in the Department of Engineering Mathematics at the University of Bristol. Upon completing his Ph.D. and subsequent post-doctoral work in Bristol, he joined the largely experimental group of Dr S.J. Elston in the Department of Engineering Science at the University of Oxford. This close collaboration with experimentalists significantly extended Dr Mottram's knowledge of the physics of liquid crystals and led to a large number of research articles on a wide range of liquid crystal phenomena. Dr Mottram's continued research into the theory of nematic and smectic liquid crystals has led to many new collaborations with industrial and academic researchers, the award of the British Liquid Crystal Society's Young Scientist Award 2000 and in September 2001 he became an EPSRC Advanced Research Fellow. His work in this field has also gained recognition from the International Liquid Crystal Society, being invited to become a member of the ILCC Scientific Committee.

Dr Mottram's research has befitted from a constant link with the Sharp Laboratories of Europe in Oxford. Through frequent meetings at Sharp with Dr M.J. Towler, Dr H.G. Walton and Prof E.P. Raynes (now at the University of Oxford) and through the funding (together with the EPSRC) of his research whilst in Oxford, his work has remained relevant to the display manufacturing industry. More recent is his collaboration with Hewlett Packard Laboratories in Bristol who are funding a CASE student (started October 2000). As with his collaboration with Sharp, frequent meetings with researchers in the Liquid Crystal Devices Group at HP will be of immense benefit.

The Mathematics Department at Strathclyde University has many permanent lecturing staff who are active in liquid crystal research. The Department has a highly acclaimed international reputation in the development and applications of liquid crystal theories and it has extensive world-wide connections and affiliations with mathematics and physics departments (both theoretical and experimental). It also attracts many visitors from academia and industry, who particularly welcome the insights and impetus to technological advances in displays which theoretical results can provide. Research facilities at Strathclyde are first rate and computing facilities are excellent. Such an environment would be of significant advantage when undertaking the proposed research and collaboration within the group has already led to the successful application for funding from the EPSRC. The post-doctoral project *Fast, Soliton-Driven Switching in Smectic Liquid Crystal Displays* (GR/N28238) commenced in October 2000.

**Relevant publications**

[M1] N.J. Mottram and S.J. Hogan, Disclination Core Structure and Induced Phase Changes in Nematic Liquid Crystals, *Phil. Trans. Roy. Soc.*, **355**, 2045 (1997).

[M2] N.J. Mottram and S.J. Hogan, Multiple Solutions in Twisted Nematic Liquid Crystal Layers, *Cont. Mech. Thermodyn.*, **9**(4), 213 (1997).

[M3] N. Ul Islam, N J. Mottram and S.J. Elston, Stability of the formation of the Chevron Structure, *Liquid Crystals*, **26**(3), 1059 (1999).

[M4] N J. Mottram, N. Ul Islam and S.J. Elston, Biaxial Ordering at the Chevron Interface in Smectic Liquid Crystals, *Phys. Rev. E*, **60**, 613 (1999).

[M5] N.J. Mottram and S.J. Elston, Thresholdless Switching Induced by Polar Anchoring in Antiferroelectric Liquid Crystals, *Liquid Crystals*, **26**, 1853 (1999).

[M6] S. J. Elston and N. J. Mottram, Order Parameter Variation in Smectic Liquid Crystals, in *Advances in Chemical Physics*, Edited by J. K. Vij, John Wiley & Sons, (2000).

[M7] N. J. Mottram, T. J. Sluckin, S. J. Elston and M. J. Towler, Shear induced melting of Smectic A Liquid Crystals, *Phys. Rev. E*, **62**, (2000).

# Part 2: Description of the Proposed Research and its Context

## 1 Background

Grating aligned nematic liquid crystal (LC) cells are of interest for displays and other optical devices [10, 11]. To understand and optimise these devices one needs to be able to probe the LC director profile around and above the grating structure. No one is able to do this at present.

Work at Hewlett Packard Laboratories, Bristol (HPL) is focused on developing a technique that can, using cells without gratings, characterise different surface treatments and LC materials and that can ultimately be used to deduce the director structure in grating aligned cells. If the method is to be useful for testing a wide range of materials and device configurations, it must be experimentally simple and the algorithms for solving the inverse problem must require much less manual intervention and give results much more quickly than is currently possible.

The LC problem is closely related to other electromagnetic problems at lower frequencies. In particular the use of low frequency electrostatic and electromagnetic measurements for location, detection and imaging problems.

The low frequency case is relevant to work at Sensatech which focuses particularly on the development of capacitance sensors for the location of anti-personnel land mines with low metal content.

### 1.1 Electromagnetic Inverse Boundary Value Problems

The general inverse problem of recovering electromagnetic properties in the interior of a body from measurements at the boundary covers a diverse range of applications. For low frequency electromagnetic fields the Caldéron [1] problem of recovering a conductivity (or permittivity) from the Dirichlet-to-Neumann mapping is the archetypal scalar inverse boundary value problem with applications to geophysics, medical imaging, non-destructive testing and industrial process monitoring. The theory of uniqueness of solution and (in)stability as well as reconstruction algorithms have been studied intensively for the last two decades [2]. More generally, in the frequency domain the inverse boundary value problems for Maxwell's equations

$$\begin{aligned} \nabla \times \mathcal{E} &= -i\omega\mu\mathcal{H} \\ \nabla \times \mathcal{H} &= i\omega\varepsilon\mathcal{E} \\ \nabla \cdot \mu\mathcal{H} &= \nabla \cdot \varepsilon\mathcal{E} = 0 \end{aligned}$$

are less well understood mathematically, although there are some uniqueness results for recovering scalar material parameters [5] [6] [7]. Both scalar and vector inverse problems can be generalised to the case of anisotropic material properties; for the scalar case there are some uniqueness results [3] [4], whereas in the vector case, there is very little known and there is no significant numerical work published.

### 1.2 Determining nematic liquid crystal director angles

The main focus of this project, which was conceived at a recent Smith Institute Faraday Workshop[1], is the inverse problem of determining nematic liquid crystal (LC) director angles from optical measurements. This is a special case of the determination of an anisotropic permittivity tensor $\varepsilon$ in Maxwell's equations. There are a number of features of this problem which make it an ideal special case of the full anisotropic Maxwell's inverse problem to explore. The first important feature is that while the permittivity tensor is anisotropic, the magnetic permeability is essentially isotropic and uniform. This means that the diffeomorphism invariance of the boundary data for Maxwell's equations is not a problem [8]. A second important feature is that the permittivity, to a reasonable approximation, varies only in the direction of its one distinguished eigenvector (the director). This means that there are only two unknown functions to recover (the two Euler angles for the director) rather than the six for a general permittivity tensor. A third mathematical attraction of this problem is that models of liquid crystals predict that the permittivity satisfies an elliptic system of equations (see Section 1.2.4). This gives a strong *a priori* smoothness condition on the solution of the inverse problem, which may be enough to make the problem well posed, or at least reduces the ill-posedness. This smoothing constraint can be explicitly incorporated in the reconstruction algorithms using regularisation. Moreover from an experimental viewpoint, the problem has the advantage over many low frequency electromagnetic imaging problems that a relatively large and accurate data set can be collected from a well developed experimental methodology.

#### 1.2.1 Optical measurements

The experimental set-up at HPL uses a commercial (Thor Labs) polarimeter to measure the Stokes parameters [12] of the light transmitted by a test cell as a function of the incident angle and polarisation state of the input beam.

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[1] http://www.smithinst.ac.uk/events/InverseProblems

Taking the $z$-axis in the direction of propagation and defining the $x$ and $y$-axes relative to the laboratory system, the electric field can be written $\mathcal{E} = (\mathcal{E}_x, \mathcal{E}_y)^t$ with $\mathcal{E}_x = a\cos(kz - \omega t)$ and $\mathcal{E}_y = b\cos(kz - \omega t + \delta)$. The Stokes parameters are then defined as

$$S_0 = a^2 + b^2,\ S_1 = a^2 - b^2,\ S_2 = 2ab\cos\delta \text{ and } S_3 = 2ab\sin\delta$$

### 1.2.2 Forward problem

For cells without gratings, HPL use the Berreman 4 × 4 matrix method [13] to solve the forward problem. This divides the cell into a number of uniform bi-refringent layers. For each layer a transfer matrix can be calculated (matching the field vector $(\mathcal{E}_x, \mathcal{E}_y, \mathcal{B}_x, \mathcal{B}_y)^t$ at each boundary) and for $m$ layers we have

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{B}_x \\ \mathcal{B}_y \end{pmatrix}_{out} = F_m F_{m-1} \dots F_2 F_1 \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{B}_x \\ \mathcal{B}_y \end{pmatrix}_{in} = \mathbf{F} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{B}_x \\ \mathcal{B}_y \end{pmatrix}_{in}$$

where $F_j$ is the transformation matrix of the $j$th layer. Given the polarisation state of the input wave, the polarisation states of the reflected and transmitted waves can be calculated. This method has the advantage over the simpler 2 × 2 extended Jones method [14] used elsewhere in that reflected waves are also included.

For complex LC display configurations, e.g.those with bi-directional grating substrates, a simple layered model cannot be used. Finite element methods will be used in the solution of the forward problem and the formulation of the linearised inverse problem. This extends considerable work by Lionheart and others on scalar isotropic inverse boundary value problems (see for example[L6],[L5]). In collaboration with engineers at Lancaster and Corus Group, work has already begun on a simplified isotropic vector electromagnetic problem of imaging molten metal using induced eddy currents. For this problem, edge finite element software is being developed for the solution of the forward problem. Edge elements, more generally Whitney elements [25], provide an elegant and practical way to solve electromagnetic problems involving vector fields. The small scale of the liquid crystal cells under consideration make these Whitney finite elements eminently feasible.

### 1.2.3 Inverse problem

Even the simpler problem of probing cells without gratings to obtain information about surface interactions and LC properties is not straightforward. Existing measurement techniques rely on a variety of strong assumptions (about the LC properties, the cell thickness, the surface orientation angle etc.). Several groups [21, 22, 23, 24] have overcome these difficulties by measuring transmission and reflection data for a test cell and then solving the resulting inverse problem (although their fitting process is not couched in these terms). Their fitting procedure is largely manual and it can take weeks to achieve an acceptable fit. A tailored fitting method which uses the *a priori* smoothness conditions and a fast method for calculation of the Jacobian would result in a dramatic speed-up.

For simulated Stokes parameter data, solutions to the inverse problem can be obtained. There are many unanswered questions. For example: How many incident angle and input polarisations are needed to give a reliable solution? Does extra data improve the solution, or make it easier to find? How can the solution of the inverse problem be reliably automated?

For grating aligned structures, where a layer model cannot be used, the problem is to find the permittivity tensor throughout the LC material. As the ordinary and extraordinary refractive indices can be measured directly, the inverse problem is to find the two Euler angles of the director as a function of three space variables.

While there are many finite element programs available commercially to solve electromagnetic problems, experience has shown that these cannot be adapted for use in solving inverse problems without access to the source code. The assembly of the finite element system matrix requires the calculation of integrals of basis functions over elements. The same integrals appear in the assembly of the Jacobian matrix which is used for the solution of the linearised inverse problem. Thus, the assembly of the system matrix and the Jacobian are closely coupled. Moreover, typical commercial programs are optimised for a problem with fixed material parameters and one set of boundary data, whereas in inverse boundary value problems one needs to solve for many different sets of boundary data. The forward problem has to be re-solved as the interior material properties are updated.

### 1.2.4 Liquid crystal models

In all ill-posed problems it is important to include all *a priori* information in the reconstruction. This can include both static and dynamic models for the material parameters which are sought. Where the material parameters themselves are known to satisfy an elliptic system of PDEs this implies interior regularity. In the LC case the system of PDEs is known but the boundary conditions are known imprecisely.

For nematic materials the continuum theory description given by the Ericksen-Leslie equations [17, 18] are now considered to accurately describe the static and dynamic behaviour of nematics. These equations take the form of a set coupled nonlinear partial differential equations governing the linear and angular momentum of the system. They are essentially the Navier-Stokes equations with the normal Newtonian fluid stress tensor replaced by an anisotropic tensor dependent on the director orientation. Coupled to the fluid flow equation is an equation governing the rotation of the director.

For static situations this system of equations reduces to two coupled nonlinear elliptic partial differential equations governing the two Euler angles describing the director orientation. In this situation the governing equations are equivalent to the Oseen-Frank equations for minimisation of elastic energy[19]. The boundary conditions at the cell substrates are usually taken to be either Dirichlet when the anchoring at the alignment layer is strong or a mixed condition when it is assumed that a surface torque balance equation results from the balance of alignment and elastic torques.

To investigate the switching process, measurements are taken with voltages applied to the cell. When a voltage is applied across the LC display the electric potential, which couples to the director orientation through a dielectric effect, is found by solving the electrostatic equations in addition to the director angle equations.

With voltages applied, it will also be important to include the flexoelectric effect [19] which is thought to play a significant role in the switching of some LC materials.

The Ericksen-Leslie equations coupled with electrostatic Maxwell's equations have been successfully used to model many different liquid crystal phenomena but are generally simplified in order to make them analytically tractable. However, numerical solutions of the full equations for static problems are readily obtained by standard numerical techniques. Hence we can reformulate the inverse problem as the recovery of the behaviour near the boundary of the director field.

### 1.3 Extension to mine detection and other inverse permittivity problems

Once a Whitney Element program has been developed for the liquid crystal problem, that code could be used for longer wavelength problems for larger domains. At the Faraday Workshop, a synergy was noticed between the LC problem presented by Hewlett Packard Laboratories and the electromagnetic detection of land mines problem presented by Sensatech. In a joint project with QinetiQ, Sensatech are developing electrostatic and electromagnetic detectors for plastic anti-personnel mines for civilian mine clearance. The contrast between mines and soil is in the permittivity and, in the case of damp ground, in conductivity as well. Where relatively low frequency electromagnetic fields are used the magnetic field can be neglected resulting in a scalar forward problem. However, there are considerable advantages in using a complementary formulation [9, 20] which, for three dimensions, results in a vector system of partial differential equations. Here the Whitney element program developed for the LC problem could be adapted to the complementary low frequency problem. For electromagnetic measurements the program could be used more directly, with modified boundary conditions.

## 2 Programme and Methodology

The programme of work is divided in to 5 Work Packages whose methodology is detailed below, together with milestones against which progress will be measured.

**WP1** The first task will be to apply established techniques for the calculation of the Fréchet derivative of the Berreman model with respect to the director angles and to derive an efficient method for the calculation of the Jacobian matrix. This can then be used to produce a fast algorithm for reconstructing the director profile in the essentially one dimensional case where the director angle is assumed to be a function $z$ only. Prior smoothness information can be incorporated using standard regularization techniques. The results of this algorithm can be validated against simulated data and against experimental data from HPL. The implementation and validation of this algorithm will be **Milestone 1**.

**WP2** The development of a Whitney element program for the solution of the anisotropic time harmonic Maxwell equations can be begun in parallel with WP1. Indeed, by the start of this project the Inverse Problems group at UMIST will have considerable experience in this area from the development of the edge element code for eddy current imaging. The extension of this code to the more general anisotropic case is easily underestimated. In anisotropic problems careful attention must be paid to the mesh generation as this can constrain the anisotropic permittivity unintentionally. The resulting code can be validated against commercial FEM codes (for example FEMLAB), and against the Berreman model for the layered case. Finally, the code needs to be validated against experimental data from HPL. Commercial mesh generation codes (such as FEMLAB), as well as free ones (such as NETGEN), can be used to generate suitable tetrahedral meshes respecting the grating surface. Implementation of the Whitney element forward solution will be **Milestone 2**.

**WP3** The Jacobian can be calculated efficiently using an integral formulation. For a change $\delta\epsilon$ in the permittivity tensor the change in the normal boundary component of the Poynting vector $\delta(\mathcal{H} \times \mathcal{E}^*)$ is given [6] by

$$\iiint \delta\varepsilon\mathcal{E} \cdot \mathcal{E}^*$$

The polarization formula can be used to derive the derivative of a boundary measurement of the transmitted field on the face opposite the incident light. The chain rule can then be used to find the derivative of the Stokes parameters with respect to the director angles. The numerical calculation of the Jacobian will be validated numerically by perturbation calculations with the forward model.

Synthetic data will be generated using the forward model, and simulated noise added. Reconstructions will then be performed using a regularized Newton's method. Implementation and validation of a regularized Newton's based reconstruction algorithm on artificial simulated data represents **Milestone 3**. At this stage collaboration with Sensatech will begin (see WP5).

**WP4** A careful investigation of models for nematic liquid crystals (see Background) will be undertaken and the implications of these models for the inverse problem. The theoretical impact of the *a priori* smoothness of the permittivity tensor satisfying the Ericksen-Leslie equations will be investigated by Lionheart in collaboration other mathematicians specialising in stability of inverse problems (Kurylev and collaborators at Loughborough and Alessandrini in Trieste). It is anticipated that stronger stability estimates can be derived than the conditional logarithmic stability estimates known to be optimal for scalar inverse problems. The successful theoretical investigation will constitute **Milestone 4a**.

More practically, the smoothness must be incorporated in the inverse problem solution. This will be done either by using a regularizing operator as in generalised Tikhonov regularization, or by choosing a smooth set of basis functions, or by a combination of these methods. The numerical implementation of the regularizing operator could use existing codes from Strathclyde [M1], or be developed using the same Whitney finite element method, as it also a vector system of partial differential equations. Any theoretical predictions about stabilty can be compared with numerical results, or numerical results may provide insight into stability. A regularized Newton's method incorporating systematic *a priori* information will be developed and tested first on simulated data, then on experimental data. The completion of the implementation and testing will constitute **Milestone 4b**.

**WP5** The investigation of the extension of the techniques developed for the LC problem to electromagnetic detection of low metal content anti-personnel land mines will constitute WP 5. This will be a preliminary investigation in collaboration with programmers from Sensatech. Senstatech already use software developed in the EIDORS project and collaborate with Lionheart on capacitance imaging. **Milestone 5** will be a report on the use of Whitney finite elements in electromagnetic mine location. This may result in a further research proposal, a postgraduate student project or a commercial research contract.

## 3 Project Management

The Project will form part of the activity of the Smith Institute Faraday Partnership. The RA will be based in the Mathematics Department at UMIST working with Lionheart. To aid the development of *a priori* smoothness constraints using liquid crystal models he or she will spend approximately one month working for Mottram in the Mathematics Department of Strathclyde. Formal meetings will be held at Hewlett Packard Laboratories twice a year involving, Lionheart and Mottram, Dr Chris Newton of HPL and one of the Smith Institute's Technology Translators. In addition, the RA will prepare progress reports for each of the semi-annual Faraday Partnership Plenary Meetings which are held in early January and early July of each year. These meetings will provide an opportunity for the Smith Institute Scientific Committee and the Faraday Advisory Board to learn of progress and to seek clarification on the academic and industrial aspects of the project from Lionheart and Newton, who will join the Scientific Committee and Advisory Board respectively. In addition to these formal mechanisms for project management, the RA will visit HPL on average every 2 months for discussions with Newton and colleagues, as well as attending the British Workshops on Inverse Problems (3 times a year) and visiting Kurylev's group at Loughborough. A final report will summarise what has been achieved against the milestones.

## 4 Relevance to Beneficiaries

The project aims to develop a technique which rapidly and by straightforward optical measurements, gives the director orientation in a liquid crystal cell, including those with bi-gratings. This will have a major impact on the design on liquid crystal displays.

The development of inverse problem techniques for anisotropic electromagnetic parameters paves the way for the development of a variety of lower frequency (microwave and radio frequency) techniques in non-destructive testing, medical imaging, industrial process monitoring and geophysics. The offshoot development of a complementary formulation for low frequency (scalar problems) has a similar range of applications. We hope particularly to aid the development of electrostatic and electromagnetic detectors for low metal content anti-personnel mines.

In the wider context, other researchers and industrial organisations participating in the Smith Institute Faraday Partnership, as well as those who attend the British Workshops on Inverse Problems and members of the Virtual Centre for Industrial Process Tomography will benefit from exposure to the techniques that will be developed during the project.

With the publication of these results and open source code, the liquid crystal community in general will benefit greatly from this project. A technique for determining the director structure within an LC cell using only standard optical experimental methods will be an important research and development tool for physicists, engineers and material scientists in academia and industry.

# 5 Dissemination and Exploitation

At the completion of milestone 3 a dissemination event, probably a masterclass along the lines of those run by Lionheart for the EIDORS project, will be held for those interested in other electromagnetic inverse problems. A beta test version of the code will be released under an open source license.

The Faraday Partnership will enable the dissemination of research results to other researchers and organisations through its Scientific Committee, Advisory Board and Technology Translators. In addition, the Smith Institute will organise annual industrial academic meetings around each of its various core research themes, providing an open forum that will allow the exploration of further related opportunities. The currently proposed project will be an important input to meetings on the theme of Inverse Problems held jointly with the BWIP.

The results will also be disseminated through the usual channels of conference papers and publication in mathematics journals including interdisciplinary journals such as Inverse Problems and Liquid Crystals and the Optics literature.

Hewlett Packard is a leading global supplier of computing, imaging and printing products, and has a significant presence in the UK, including HP Labs Bristol which is the largest HP research facility outside Palo Alto and the main site in Europe. There is an active program at HPL Bristol developing novel display technologies for intelligent information and imaging devices, to which the results of this proposal would be immediately applicable.

Sensatech is a world leader in the design and application of capacitive sensor technology. It was founded in 1993 and is based in Brighton. The majority of their work is research and development and they have developed products for the commercial, industrial, military and medical fields. They have collaborative arrangements with a number of universities as well companies such as Clarks Shoes and BNFL. Their collaboration with the UMIST group came began with their attendance at an EIDORS masterclass in 2000, subsequent collaboration on land mine detection and through their attendance at the Faraday Partnership workshop.

The parties have signed a Heads of Agreement concerning the ownership and exploitation of foreground and background IPR from this project.

# 6 Justification of Resources

## 6.1 Staff

Given the collaborative requirements of this project, and the complexity of finite element modelling and inverse problem solution, a two-year appointment at the level of postdoctoral research assistant is sought, initially at spine point 8 to give reasonable scope for finding a candidate with the right mix of expertise. The RA should have a background in computational electromagnetics and preferably inverse problems or analysis of PDEs. The computing officer is required for the installation and maintenance of computing hardware and software to be used at UMIST, where the RA will do the bulk of his or her work. The secretary is needed for local administrative work associated with the day-to-day running of the project and dissemination of the results.

## 6.2 Travel

The travel resources have been requested in order to allow the RA to stay at University of Strathclyde and for both the RA and Lionheart to make regular visits to Hewlett Packard in Bristol. The former are needed for development of LC models and the latter for validation against experimental results, and understanding of the engineering problems and measurement accuracy. The RA will also attend inverse problems conferences each year and one international conference as part of the dissemination process. Travel costs for Mottram are covered by his EPSRC Advanced Research Fellowship.

### 6.3 Equipment

A workstation for the RA has been requested. The project will involve significant numerical simulation, including 3D finite element calculations and a machine with high speed and large memory is requested. A preconfigured Linux machine is requested for compatibility with the local network and would be supported by the local computing officer. This machine will also be needed for the preparation of papers, using the world-wide web and e-mail.

### 6.4 Indirect contributions from the collaborators

HPL will provide staff time, existing Berreman code and experimental data, Sensatech will provide staff time and experimental data. These contributions are detailed on the attached letters of support.

## References

[1] A. Caldéron, in: Semin. Numer. Anal. Appl. (Rio de Janeiro), p.65 (1980).
[2] V. Isakov, *Inverse Problems for Partial Differential Equations*. Springer, New York (1998).
[3] J. M. Lee and G. Uhlmann, Comm. Pure Appl. Math. **42**, p.1097 (1989).
[4] G. Alessandrini and R. Gaburro, SIAM J. Math. Anal. **33**, p.153 (2001).
[5] P. Ola, L. Päivärinta and E. Somersalo, Duke Math. J. **70**, p.617 (1993).
[6] E. Somersalo, D. Isaacson and M. Cheney, J. Comput. Appl. Math. **42**, p.123 (1992).
[7] M. S. Joshi and S. R. McDowall, Pac. J. Math. **193**, No.1, p.107 (2000).
[8] W. R. B. Lionheart, Lect. Notes Math., Springer, to appear (2001).
[9] A. Tamburrino, S. Ventre and G. Rubinacci, Inverse Problems **16**, p.1585 (2000).
[10] G. P. Bryan-Brown, *et al*, Proc. IDRC, SID ’94, p.209 (1994).
[11] G. P. Bryan-Brown,*et al*, Proc Internation Displays Research Conference (SID), p.37 (1997).
[12] B.A. Robson, *The Theory of Polarisation Phenomena*, Clarendon Press, Oxford (1974).
[13] D. W. Berreman, Liquid Crystals **62**, p.502 (1972).
[14] A. Lien, Optical Society of America **22**, p.171 (1997).
[15] C. V. Brown, G. P. Bryan-Brown and V. C. Hui, Proc. 16th ILCC, (1997).
[16] C. J. P. Newton and T. P. Spiller, Proc International Displays Research Conference (SID), p.13 (1997).
[17] J. L. Ericksen, Arch. Rat. Mech. Anal. **4**, p.231 (1960).
[18] F. M. Leslie, Quart. J. Mech. Appl. Math. **19**, p.357 (1966).
[19] P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals*, 2nd ed. OUP, Oxford (1993).
[20] D.M Spink, COMPEL 15 (4): 63, 1996
[21] D.Y.K. Ko and J. R. Sambles, J. Opt. Soc. Am., A**5**, 1863 (1988).
[22] N.J. Smith and J. R. Sambles, J. Appl. Physics, **85**, p.3984 (1999).
[23] B. T. Hallam, F. Yang and J. R. Sambles, Liquid Crystals, **26**, p.657 (1999).
[24] A. Mazulla, F. Ciuchi and J. R. Sambles, Phys. Rev. E, **64**, p.21708 (2000).
[25] A Bossavit, Computational Electromagnetism.*Variational Formulation, Complementarity, Edge Elements*, Academic Press, San Diego (1998).

Liquid crystals (LCs) are a state of matter which has properties between those of conventional liquids and those of solid crystals. For instance, a liquid crystal may flow like a liquid, but its molecules may be oriented in a crystal-like way. There are many different types of liquid-crystal phases, which can be distinguished by their different optical properties (such as textures). The contrasting areas in the textures correspond to domains where the liquid-crystal molecules are oriented in different Electromagnetic inverse problems for nematic liquid crystals and capacitance imaging. Nick Polydorides.Â  The inverse problem of reconstructing the orientation of the director vector in a uniaxial nematic liquid crystal using a finite set of noise infused boundary polarization measurements is approached as a special case of the inverse permittivity tensor problem, where the dielectric tensors are symmetric and expected to vary most significantly along the directions of their two biggest eigenvalues, which correspond to the.