Kähler Geometry of Loop Spaces

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http://gonitsora.com/kahler-geometry-loop-spaces/

Armen Sergeev

Mathematical Society of Japan, 2010, 228 pp.

This book concerns Kahler geometry and the geometric quantisation of loop spaces. The main objects are three important examples of infinite Kahler manifolds: loop spaces of compact Lie groups, Teichmuller spaces of complex structures on loop spaces, and Grassmannians of Hilbert spaces.

It consists of five parts: Part I. Preliminary Concepts. There are six chapters in this part for reviewing all necessary background material. Chapters 1 and 2 give a quick introduction to Frechet manifolds and Frechet Lie groups. The readers who are familiar with the basic knowledge of differentiable manifolds can easily follow the contents. Chapter 3 contains basic facts on flag manifolds and irreducible representations of semisimple Lie groups. The central extensions and cohomology of Lie groups and Lie algebras are reviewed in Chapter 4. The Grassmannians of a Hilbert space are discussed in Chapter 5. In Chapter 6, quasiconformal maps and their basic properties are reviewed.

Part II. Loop spaces of compact Lie groups. Various geometric properties of the loop space G of a compact Lie group G are discussed, such as symplectic structure, complex structure and Kahler structure. A canonical embedding of flag manifolds of a Lie group G into G is described and the Grassmannian realisation of G is constructed. The central extensions of loop groups and loop algebras are also studied. There are three chapters (7–9) in this part.

Part III. Spaces of complex structures. This part is devoted to various spaces of complex structures on the loop groups G. It consists of two chapters. Chapter 10 is about the Virasoro group and its coadjoint orbits, and Chapter 11 is about universal Techmuller space.

Part IV. Quantisation of finite dimensional Kahler manifolds. This is a brief introduction to the geometric quantisation of finite dimensional Kahler manifolds, which consists of three chapters (12–14) concerning Dirac quantisation, Kostant–Souriau presentation and Blattner–Kostant–Sternberg quantisation.

Part V. Quantisation of loop spaces. After solving the geometric quantisation problem for the loop space of a d-dimensional vector space in Chapter 15, a geometric quantisation of the loop space G of a compact Lie group G is constructed in Chapter 16. This part provides nice representations and twister quantisation of loop groups.

The study on the geometry and topology of loop spaces is motivated by the relation of these spaces and various problems in modern mathematical physics, such as string theory. This book gives a good introduction to the Kahler geometry of loop groups. Various geometric properties of the loop groups are explored in a concise way with all necessary concepts reviewed. In addition to geometers, this book could be a good reference for topologists. The geometric properties of loop groups may admit applications to the homotopy theory of loop groups. This book can be selected as a main reference in one-semester topic courses. For graduate students in geometry and topology, this book can be listed as one of basic references.

Jie Wu.

National University of Singapore.

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[1] Baraglia, D. and Huang, Z., â€˜Special KÃ¤hler geometry of the Hitchin system and topological recursionâ€™, Preprint, 2017, arXiv:1707.04975. [2] Eynard, B., â€˜A short overview of the â€œTopological recursionâ€â€™, Preprint, 2014, arXiv:1412.3286. [3] Eynard, B. and Orantin, N., â€˜Invariants of algebraic curves and topological expansionâ€™, Commun. Number Theory Phys. 1(2) (2007), 347â€“452. [4] Freed, D. S., â€˜Special KÃ¤hler manifoldsâ€™, Comm. Math. Phys.Â [6] Hitchin, N. J., â€˜Metrics on moduli spacesâ€™, Contemp. Math. 58 (1986), 157â€“178.

The based loop group, ΩG, is known to admit a Kaehler metric, given as \begin{equation} g(X,Y)=2\sum_{k>0}k\textrm{Tr}(X_{-k}Y_k), \end{equation} this is given in page 150 of Segal and Pressley's 'Loop Groups', where $X,Y\in L\mathfrak{g}$, the loop algebra. ΩG can be described as the homogenous space LG/G, and there is a transitive LG isometry on ΩG. It is known that the above metric can be shown to be LG-invariant, see e.g., Khesin and Wendt's 'The Geometry of Infinite Dimensional Groups', page 239, below Corollary 4.8, and Armen...Â An additional reference which is useful is Dan Freed's 'The Geometry of Loop Groups'. [1] Baraglia, D. and Huang, Z., â€˜Special KÃ¤hler geometry of the Hitchin system and topological recursionâ€™, Preprint, 2017, arXiv:1707.04975. [2] Eynard, B., â€˜A short overview of the â€œTopological recursionâ€â€™, Preprint, 2014, arXiv:1412.3286. [3] Eynard, B. and Orantin, N., â€˜Invariants of algebraic curves and topological expansionâ€™, Commun. Number Theory Phys. 1(2) (2007), 347â€“452. [4] Freed, D. S., â€˜Special KÃ¤hler manifoldsâ€™, Comm. Math. Phys.Â [6] Hitchin, N. J., â€˜Metrics on moduli spacesâ€™, Contemp. Math. 58 (1986), 157â€“178. Every Banach space is a FrÃ©chet space, as the norm induces a translation-invariant metric and the space is complete with respect to this metric. The vector space Câˆž([0, 1]) of all infinitely differentiable functions Æ’: [0,1] â†’ R becomes a FrÃ©chet space with the seminorms. â€– f â€– k = sup { | f (k) (x) | : x âˆˆ [0 , 1] } {\displaystyle \|f\|_{k}=\sup\{|f^{(k)}(x)|:x\in [0,1]\}}. for every non-negative integer k. Here, Æ’(k) denotes the k-th derivative of Æ’, and Æ’(0) = Æ’. In this FrÃ©chet space, a sequence (Æ’n) of functions converges towards the element Æ’ of Câˆž([0, 1]) if and only if for every ...Â KÃ¤hler Geometry of Loop Spaces. Mathematical Society of Japan Memoirs. 23. World Scientific Publishing.