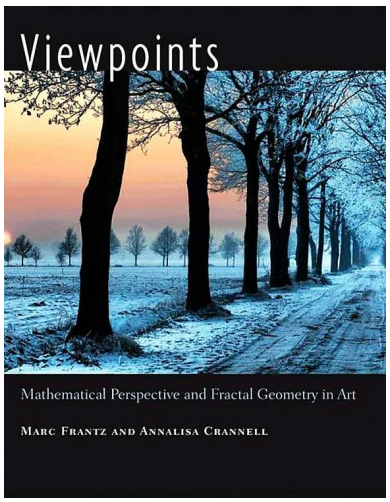
Book Reviews

Viewpoints: Mathematical Perspective and Fractal Geometry in Art

Marc Frantz and Annalisa Crannell
Princeton University Press, 2011, ISBN: 978-0-691-12592-3

This is a textbook on mathematical methods for undergraduate students in art and design, probably unique in its genre. The authors have two aims: the first is to overcome the traditional math anxiety of art students and the second is to acquaint them with two mathematical topics that are useful in producing convincing graphical representations: perspective for the built environment, and fractal geometry for the natural environment.

The first objective is addressed by including a series of personal essays which the authors call Artist Vignettes. These are written by seven guest authors whom the book's authors call professional artists, but who are, for the most part, academics in departments of fine art in US universities. The vignettes include statements about the influence of mathematics on the authors' formation and practice as well as colour plates displaying some of their work. Apart from the book cover shown below, the illustrations have little bearing on the mathematical topics of the book.



The two mathematical sections of the book are quite distinct in approach. The first section, on monocular linear perspective, is cookbook in style. The authors present rules (even a spreadsheet) for using similar triangles to represent projections of three-dimensional geometric objects, usually buildings, onto a two-dimensional picture plane. Conversely, they show how to calculate from such an image the position of the eye or camera and the position and dimensions of the depicted objects. Other topics considered as extra reading material are spherical perspective and anamorphic art.

Such a workshop approach may work well for the intended audience. However, I believe the objective of overcoming distaste for mathematics could have been better addressed by adopting a more historical approach. Surely the subject of perspective is a wonderful opportunity to demonstrate the debt that mathematics owes to art. This is not mentioned in the text, and the twelve colour plates contain no example of historically significant paintings in which perspective plays a major role.

A second criticism concerns the authors' explanation of what they call the 'skyscraper paradox'. This concerns the apparent convergence of the sides of the front

face of a tall building to a vanishing point even though this face is parallel to the theoretical picture plane. The authors' obscure four-page explanation seems to attribute this phenomenon to some sort of optical illusion. But the real explanation is quite simple: as the eye or camera pans upward, the picture plane tilts forward and so is no longer parallel to the face of the building.

The second half of the book concerns fractal geometry. I believe that students will find this section more interesting because it is novel and requires no lengthy calculations. The short history, mainly Lewis Fry Richardson's observations on measuring coastlines and Benoit Mandelbrot's mathematical model of certain natural phenomena, is well treated. The authors also trace the notion of features of a painting being continuously repeated at finer scales to classical Chinese ink on silk paintings and 18th-century Japanese woodprints.

Some of the mathematics in this section would be formidable to the intended students: it includes both the Sierpinski gasket and carpet, the Koch curve, iterated functions and fractal dimension. I imagine that in practice, these topics would be treated cursorily, and the emphasis would lie rightly on studying the fractal aspects of the photographs of Ansel Adams and the architecture of Frank Lloyd Wright, both of which are covered in the text.

In spite of the criticisms above, this book demonstrates clearly why artists need to know some mathematics and it should appeal to art students and their teachers.

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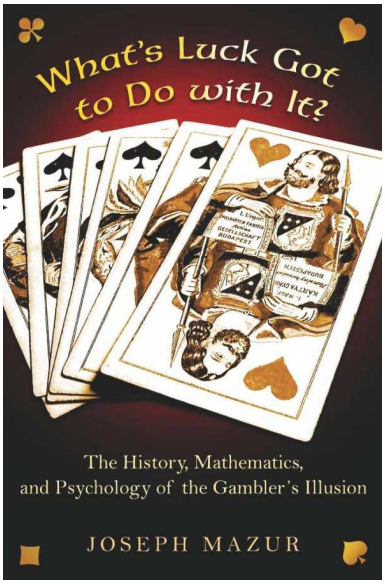
What's luck got to do with it? The history, mathematics, and psychology of the gambler's illusion

Joseph Mazur
Princeton University Press, 2010, ISBN: 978-0-691-13890-9

Gambling is a significant part of Australian life. In sport, its influence has expanded from racing horses and dogs to betting on football, cricket, and even tennis. Debate rages in places such as Bendigo and Castlemaine whenever a business applies to install more poker machines. Recently, at the national level, policies on poker machines have featured in political debates.

Australia in 2012 is not alone in experiencing the gambling phenomenon. The book by Mazur reflects on the history, mathematics and psychology of gambling. It helps us to put these issues in a broader context.

In the section on the history of gambling, Mazur reminds us that gambling has always been with us. In the Hebrew Bible, the Psalmist writes 'They part my garments among them and cast lots upon my vesture' (KJV, Ps 22.18). The Romans, who had a god for almost everything, attributed fate or luck to the goddess Felicitas. Despite occasional efforts by authorities to outlaw gambling, the pastime has persisted through the coffee houses of London, resorts on the Continent, saloons in the wild-west, and our own two-up.



Gambling can take many forms. We now have internet gambling. Many would argue that investing in financial markets is, in essence, a form of gambling. Hence our superannuation schemes involve some speculation and risk. Mazur asks 'And what about insurance? Is that not a form of wagering with a hefty hedge?' (p. 80). This may make readers stop in their tracks; after all, taking out insurance is considered wise, whereas gambling is foolish.

The second part of the book deals with the mathematics of gambling. Mathematics did not have much to contribute to gambling until relatively late in the piece. Gambling had been around for many centuries prior to the publication, in 1713, of *Ars Conjectandi* by Jacob Bernoulli. It was this work that first formulated the weak law of large numbers and is regarded as a foundation stone of probability theory. Bernoulli's work is motivated by a desire to model games of chance in terms of mathematics. If gambling were not so popular, would the theory of probability have been developed?

Mazur is writing for a wide audience. He introduces the concepts of probability distribution, binomial and normal distributions, expectation, and the weak law of large numbers. Most of this part of the book is easy reading for readers of the *Gazette*. However it's hard enough teaching the basic ideas of probability to university students, so the author has a challenging task here. Naturally, he introduces the ideas in the context of gambling. There is a glossary of mathematical terms at the back of the book. University students studying probability and statistics might also find the book an enjoyable read. It may give them some better understanding of the ideas they meet in their text books. However, not all students may be familiar with common gambling games.

The final section of the book deals with the psychology of gambling. Mazur recounts stories from novels and his own personal experiences to illustrate why, whether winning or losing, gamblers seldom know when to quit. His childhood account of a chase to win back his favourite lucky aggie shooter marble was funny, if not pitiful; he ended up losing all his marbles (literally) and some other prized souvenirs. As he noted 'Chasing losses to break even is the gambler's peril' (p. 166).

Even in games involving some element of skill and memory (e.g. gin rummy), gamblers may be overconfident and lose all in the game that mattered. More often than not, gambles are biased in favour of the 'house'. This slight bias of a few percentage points is all the casinos need to be profitable, either across the thousands who gamble there or across the number of visits by an individual gambler. Indeed, there is a saying in the casino business: 'We love to see you win, we are only afraid you will not come back'.

The fact is that we humans do not think and act like computers. Many fail to appreciate the statistical independence of each roulette spin. And even when this is acknowledged, it is amusing that roulette players studiously record the pattern of past spins (e.g. quadrants, red/black, odd/even) to predict the outcome of the next spin. This faulty logic fosters an illusion of control that is indisputably evident in poker machines where players perform rituals (e.g. rubbing the screen, or pressing some buttons in a sequence) in attempts to influence chance events. Furthermore, 'Casino corporations have researched gambling preferences and habits to learn which games, machines, and environments encourage further play . . . . One wonders why they don't give just give away a few coins to hook punters' (p. 167). Indeed they do, in the form of free meals after the patron has chalked up sufficient credits in the visit.

Humans have a personality which, in some cases, predisposes them to risk-taking activities. And they also have a memory of past successes and failures in the face of uncertain events. Their perceived value of a risk depends on a number of factors (e.g. size of the risk, personal circumstances, past history, and present situation). All these factors affect clear logical thinking. For example, contestants in the TV program 'Deal or No Deal' may be caught up by the moment (e.g. attention of the crowd, recent choices), and sometimes they fail to accept an offer (deal) that is higher than the expected value of the outcome.

Mazur charts a careful course in search of the causes of problem gambling, through early psychoanalytic theories (pleasure-seeking, driven by guilt) to modern behaviouristic theories (classical and operant conditioning) and yet some more (excitement, escape from boredom). A montage emerges of a pleasure-seeking guilt-ridden pathological gambler, conditioned to seek excitement and repeat the same mistakes over and over again. It is difficult to differentiate between factors that are causes and those that are effects, and between factors that initiate problem gambling and factors that maintain it. Mazur reminds us that there is 'no one-size-fits-all theory' (p. 198). Some factors predominate in gamblers who play poker machines, and others in gamblers who go to the dog races.

We are accustomed to cause and effect in our lives. If we drive fast, we are more likely to have a car accident. Superstition behaviour arises when this correlation between cause and effect is very low or when the cause is wrongly attributed (e.g. Mayan human sacrifice to please the 'gods'). In games of chance, on the other hand, outcomes have no underlying cause and that is where human thinking comes unstuck. Some gamblers, misguided by a recent string of red outcomes on a roulette wheel, think that another red is likely (i.e. law of small numbers). Other gamblers, misguided by the belief that a short sequence of outcomes will correct

itself in the long run, will think that a black is due (i.e. law of large numbers). No other game is as foreign to human thinking as playing the poker machine where outcomes are determined randomly, constrained by a program to deliver a house percentage in the long run. What else can we do to master this aspect in our lives? Depend on luck. We try to sense it, to ride it, and to manipulate it. As Mazur surmises, we gamble because it is intrinsically human to 'desire to manipulate luck in order to validate life' (p. 216).

Mazur's book *What's luck got to do with it? The history, mathematics and psychology of the gambler's illusion* would be a useful addition to any university library. It provides points for discussion and references for students of applied probability, psychology, education and the social sciences. The book is well bound and meets the usual high standard that we have come to expect from Princeton University Press.

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Fractals in Art. 4. Cellular Automata and Fractal Evolution. 4A.Â Fractal geometry is a new way of looking at the world; we have been surrounded by natural patterns, unsuspected but easily recognized after only an hour's training. 1. Introduction to Fractals and IFS is an introduction to some basic geometry of fractal sets, with emphasis on the Iterated Function System (IFS) formalism for generating fractals. In addition, we explore the application of IFS to detect patterns, and also several examples of architectural fractals. 2. Natural Fractals and Dimensions presents a method of measuring the complexity of fractals. Generalizing the familiar notion o Viewpoints focuses on two mathematical areas: perspective related to drawing man-made forms and fractal geometry related to drawing natural forms. Investigating facets of the three-dimensional world in order to understand mathematical concepts behind the art, the textbook explores art topics including comic, anamorphic, and classical art, as well as photography, while presenting such mathematical ideas as proportion, ratio, self-similarity, exponents, and logarithms. Straightforward problems and rewarding solutions empower students to make accurate, sophisticated drawings. Personal essays and The mathematical content revolves around two subjects: perspective projection and fractal geometry. The brick-and-mortar topics include 2d and 3d coordinates (Ch 1), similarity (Ch 2), central and inscribed angles (Ch 5), existence of orthocenter (Ch 6), self-similarity (Ch 8) and exponents and logarithms (Ch 9). On these, the authors build an edifice of perspective projection (Ch 1-7), with a. spacious attic of fractal geometry (Ch 8-9). Perspective projection is covered gently and at a considerable length. One-, two-, and three-point perspective drawings are treated in separate chapters and